A Brief Overview of Reverberation Algorithms

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Abstract: This article provides an overview of two types of reverberation algorithms----Schroeder's model and Feedback Delay Network. Through comparisons between two models designed in Multak and TC Native Reverb plug-in, I sum up the basic features of the algorithms and put forward some insights in practical reverb model design.

I. Introduction

There are several types of digital reverberation algorithms, of which the most common types are sampling method, synthesis method and the mixture of both sampling and synthesis methods.

The sampling method is the most ideal reverb algorithm. Since the digital reverberation is a LTI system, if we manage to get the pulse response of a real environment, the reverberation of a certain sound signal can be obtained by convolution of the signal and the pulse response. Through this way the output reverberation effect is highly realistic. However, the amount of calculation is considerable even if we use the fast convolution. Sony, Yamaha have produced sampling reverberators, which are relatively costly. Currently, Sony and Samplitude both use the method— to put stereo microphone in the center of a certain place, such as concert hall, and set high-end amplifier to play testing sound— pulses and sine waves mostly. Finally they collect the sound and get the pulse response through calculations.

The synthesis method is most common in digital reverb simulators—to connect a few delay units in combinations of parallel and series to generate reverb effect. The realness of synthesis method is certainly far less than that of sampling method, but it has the advantages of convenience in calculation and ease of adjustment. Also, it can generate some amazing reverb effects which do not exist naturally. The quality of synthesis method is largely dependent on the quality of algorithm. If designed well, it can bring about excellent reverb, like the famous Lexicon and TC reverb plug-ins do.

The mixture method is the combination of the two methods above. Since in early reflection (the period when sounds begin to reflect), human ears have keen sense on the intensity and time of echoes, even slight changes can make big difference in perception. In the reverb time (the period when echoes attenuates into below 60dB of source volume), the main perception come from the envelope of attenuation and spectrum features. Therefore, the mixture method is to employ real sampling convolution in early reflection and use synthesis method to simulate the envelope of attenuation and the spectrum of reverb. Since the number of non-zero pulses in early reflection not large, hence the calculation amount is acceptable and the actual effect is satisfactory. The mixture method is very popular, for many world's top reverb processor all add some mixture element in processing the reverb.

II. The Traditional Algorithm

1. The Delay Units

(a) The comb units

The simplest delay unit is an IIR filter. $x(n)$

represents the nth input point, $y(n)$ represents the nth output point, D represents the delay time, and the delay equation is:

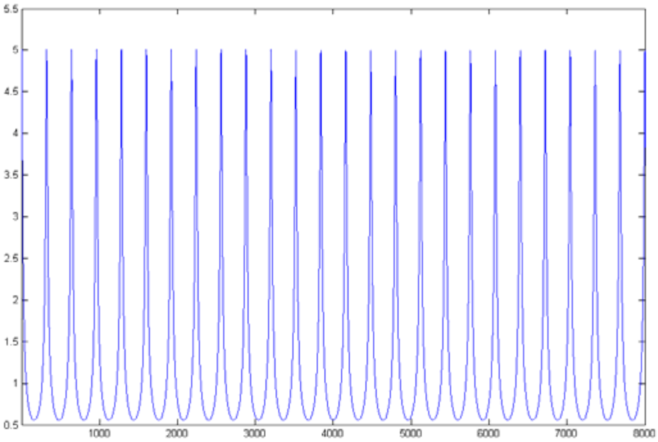
$$y(n) = x(n) + \alpha y(n - D)$$

α is the feedback coefficient.

The frequency response of the system is:

$$Y = \frac{1}{1 - \alpha z^{-D}} X$$

When $D = 100, \alpha = 0.8$, the spectrum of response under 8kHz is:



It's obvious that the response has large waves in spectrum— oscillating between 0.5 and 5 times of gain— the phenomenon of interference. Since the shape of the frequency response resembles a comb, the filter is called comb filter.

As can be observed in its spectrum, the comb unit can cause great spectral distortion, especially when comb units are in series ---- the spectrums multiply each other and doubling the effect of interference, thus causing considerable distortion and creating serious metallic sounds. However, if several comb units with different and proper delay time are connected in parallel, their corresponding spectrums can add up to make the whole spectrum flat. Therefore, the comb units are often connected in parallel.

(b) The All-pass Unit

Due to comb units' serious effect of interference, experts find delay unit of flat spectrum. We call it all-pass unit.

The system formula of all-pass unit is:

$$y(n) = g(\alpha x(n) - x(n - D)) + \alpha y(n - D)$$

α is the feedback coefficient and g is gain.

The transfer function is:

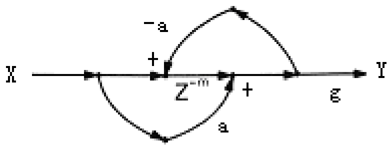
$$Y = g \frac{\alpha - z^{-D}}{1 - \alpha z^{-D}} X.$$

Apparently, all the zero and pole points of the transfer function are reciprocal, therefore the filter is an all-pass system, whose frequency response is a straight line. Also, the filter has limited distortion in phase, thus it is an ideal delay unit. Even dozens of all-pass units connected in series will not cause uncomfortable feeling in hearing. However, the weakness is difficult to control the delay effect. As can be seen its non-zero values pulse response are:

$g\alpha$, $g(\alpha^2 - 1)$, $g\alpha(\alpha^2 - 1)$, $g\alpha^2(\alpha^2 - 1)$, $g\alpha^3(\alpha^2 - 1)$……

Hence, it is not effective to control the attenuation of all-pass unit by purely increase or decrease α. When α is close to 1, although it has slow attenuation process, but $\alpha^2 - 1$ is too small that signal will be too weak after the first time delay. It is obvious that when α approach golden section point, the attenuation can be relatively proper. Generally, α = 0.7.

According to the system formula, the structure of the all-pass unit is:



The system equation is:

$$d(n) = (1-\alpha^2)x(n) - \alpha d(n-m),$$

$$y(n) = g(\alpha x(n) + d(n-m)).$$

$d(n)$ is the delay line.

If $\alpha = \frac{\sqrt{2}}{2}$, $\bar{g} = \frac{\sqrt{2}}{2}g$, $\bar{d}(n) = 2d(n)$, the equation can be written as:

$$\bar{d}(n) = x(n) - \frac{\sqrt{2}}{2}\bar{d}(n-m),$$

$$y(n) = \bar{g}(x(n) + \frac{\sqrt{2}}{2}\bar{d}(n-m)).$$

(c) Other Units

There are other kinds of delay unit, but the most important is the comb unit with low-pass filter ---- to attenuate the high frequency and make the sound warmer. Generally a comb unit is attached with a one-order low-pass filter. (both IIR or FIR is commonly used.) It is not common for all-pass unit to be designed with low-pass filter.

Multi-comb filter is another important delay unit, which can generate high-density delay effects. For example of an simple situation, the system equation as below:

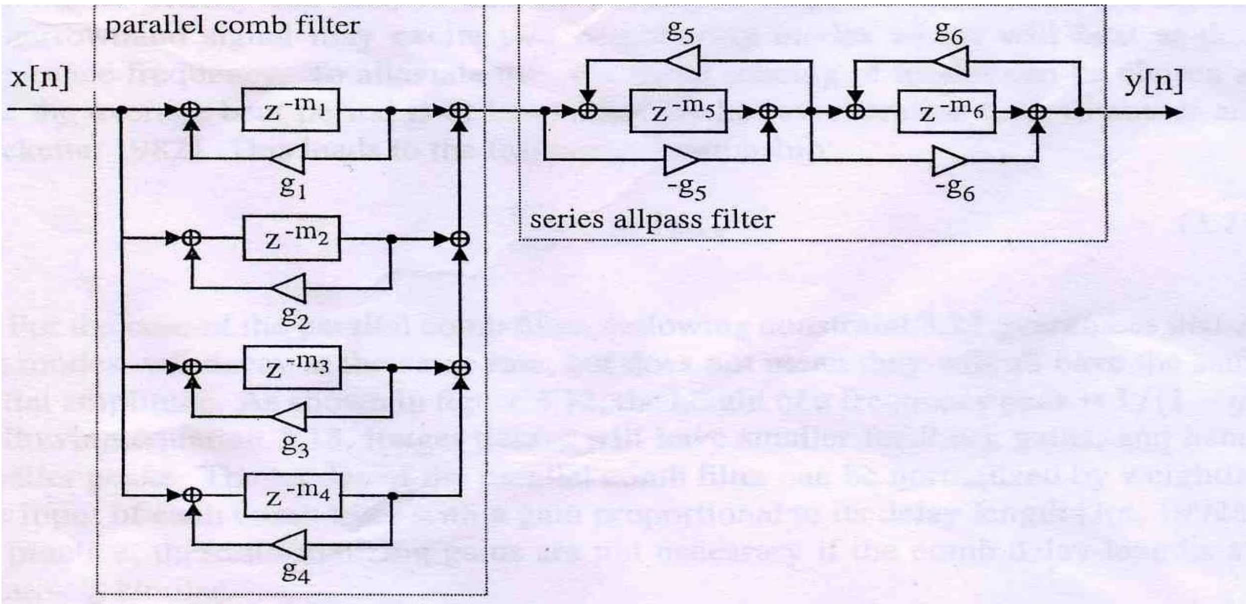
$$y(n) = x(n) + \alpha_1 y(n-D_1) + \alpha_2 y(n-D_2).$$

The unit creates an effect of delay time same as two comb units with D1 and D2 delay time connected in series, but it does not cause serious spectral distortion as comb units do. However, a multi-comb unit does not bring comfortable perception as comb units do because the attenuation of its delay does not proceed regularly.

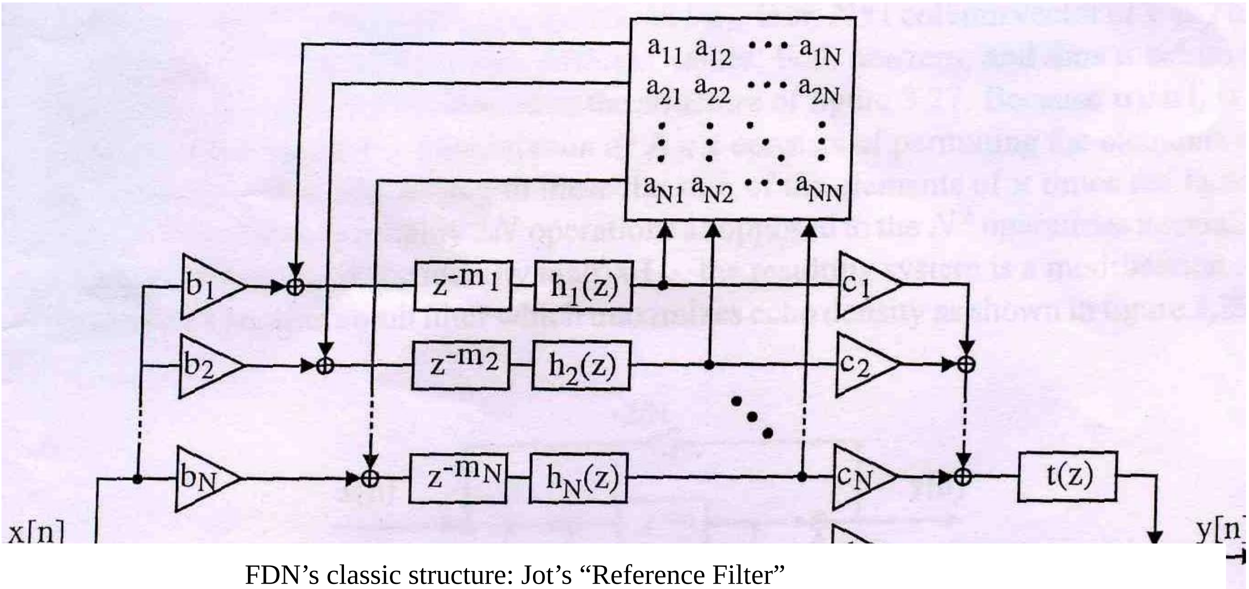
2. The Combinations of Units

To conclude according to the discussion above: comb unit is easy to control but can only be connected in parallel; all-pass filter is suitable for connecting in series to enhance depth of reverb, but uneasy to control and has weak attenuation quality. Therefore, one ideal combination is: first we connect several comb units in parallel to create a preliminary reverb with low density and good attenuation, afterwards we cascade several all-pass units to enhance depth (density). In this way we obtain a reverb effect of natural attenuation, easy controllability and good density. In fact, the most classic Schroeder and Moorer reverberation models are designed similarly. Schroeder at first cascades 5 all-pass units and later on parallels 4 comb units and then cascades 2 all-pass units to complete his model. On the other hand, Moorer connects 6 comb units with one-order low-pass IIR filter and then cascade a an all-pass unit. He thought that the interference brought by comb filter can be offset by at least 6 comb units in parallel.

Besides the structure, parameter is another deciding factor of reverberation. Every delay unit has a few parameters, such as delay time, feedback coefficient, filter coefficient, etc. The more units a model has, the more parameters need to be adjusted. As a matter of fact, every little change of any parameter can make difference in the perception of sound. Coordination of parameters is troublesome job. Moorer has provided the calculation equation of feedback coefficient with reference to delay time and reverb time. Unfortunately, few references refer to the



The classic structure of traditional algorithm: Schroeder's model



FDN's classic structure: Jot's "Reference Filter"

coordination rule of comb units. Many recent models are still following the parameters offered by Moorer and Schroeder. Parameters of most excellent reverb software are adjusted into perfection through large amounts of tests and auditions.

III. Feedback Delay Network

1. Structure

The reverberation algorithm of MF5605 is based on the FDN（Feedback Delay Network）, the digital reverberation model designed by J.M.Jot in 1997. The original MF5602 reverberation algorithm is based on the structure of several parallel comb filters and a few series all-pass filters (the traditional algorithm). In fact, FDN is derived from the traditional model. Their associations can be observed by comparison between their classic structures.

The matrix A on the top-right corner is called "Feedback Matrix", if we neglect the attached low-pass filter h_i and the correcting filter $t(z)$, the formula of the FDN structure can be written down as:

$$y(n)=\sum_{i=1}^{N} c_i s_i(n)+dx(n);$$

$$s_i(n+m_i)=\sum_{j=1}^{N} a_{i,j} s_j(n)+b_i x(n).$$

It is easy to find that if A is diagonal matrix --- $a_{i,j}=\delta(i,j)$, the FDN system is just the parallel comb filters. On the contrary, the parallel comb units of traditional algorithm can be regarded as a special FDN, whose feedback matrix is diagonal. The first step from traditional algorithm to FDN is to change the feedback matrix from diagonal to something else.

The problem is: What kind of matrix can generate most natural reverb effect?

2. The basic shape of pulse response

From the properties of reverb time, we know that after the natural reverb passes the early reflection period, it proceed into the exponential decay period with the corresponding density of pulses swiftly increases, which reflects the major two tasks of feedback matrix A---- to increase the density and control the envelope.

Experts of digital reverberation in the last century have experimented a lot to select the feedback matrix and put forward several schemes, such as circulant matrix, Householder Matrix, Hadamard Matrix, etc. Certainly, the precondition is to satisfy the "conservation of energy" （unitary, $AA^*=I$ ） More instructively, Jot has listed some principles of matrix selection, among which the most important is:

a. The matrix A should have no null coefficients, so that the recirculation through multiple delays produces a faster increase of the "echo density" along the time response.

Take the parallel comb filters as example, FDN with diagonal matrix generates reverb of low density, for which we need to connect a few all-pass filters in series to ensure the density quality(in MF5602, not only 5 all-pass units are cascaded, but elements in each channel are extracted and inserted into opposite channel as well.) Under such circumstances, more parameters have to be coordinated and it incurs the comment as "inaccuracy" to traditional algorithm from experts.

b. To speed up the convergence towards a Gaussian amplitude distribution, the "crest factor" of the matrix A (ratio of largest coefficient over RMS average of all coefficients) should be minimum. Ideally, all coefficients should have the same magnitude.

The Crest Factor of diagonal matrix is considerably big.

To sum up, the direct advantages of FDN against the traditional algorithm is: The proper selection of the feedback matrix A can swiftly increase the density of reverb time, also make the envelope more like exponential attenuation.

IV. Comparisons

1. Settings of MF5605 model

Following the two principles Jot proposed, we select an 8-order Hadamard Matrix as the Feedback Matrix:

$$A=\sqrt{2}/4\times\begin{pmatrix}1&1&1&1&1&1&1&1\\-1&1&-1&1&-1&1&-1&1\\-1&-1&1&1&-1&-1&1&1\\1&-1&-1&1&1&-1&-1&1\\-1&-1&-1&-1&1&1&1&1\\1&-1&1&-1&-1&1&-1&1\\1&1&-1&-1&-1&-1&1&1\\-1&1&1&-1&1&-1&-1&1\end{pmatrix}$$

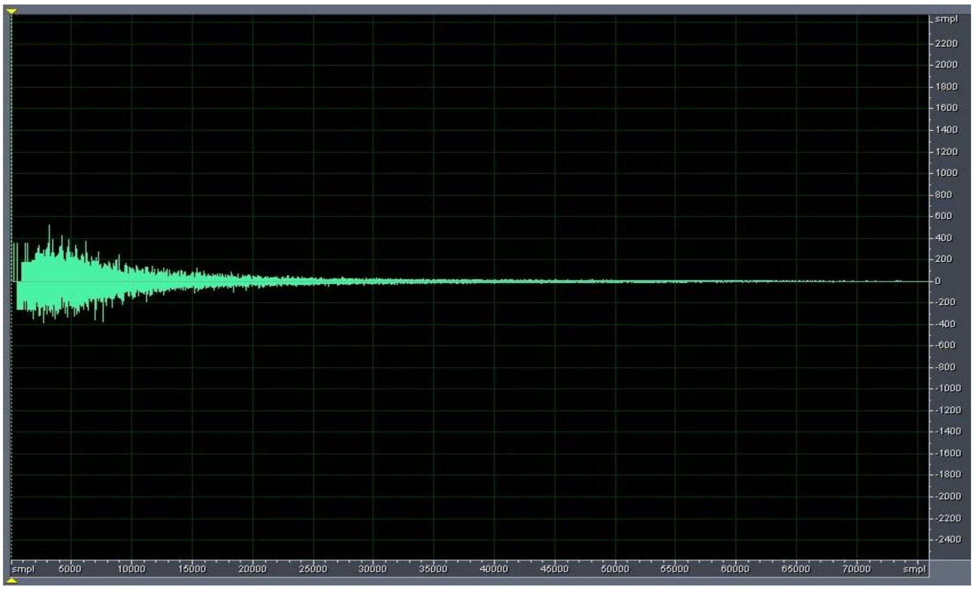
Obviously, the matrix is most reasonable among 8-order unitary matrixes regarding the two principles. We also have some adjustments to the model as follow:

a. To increase the density, connect the FDN with two all-pass filter in series.

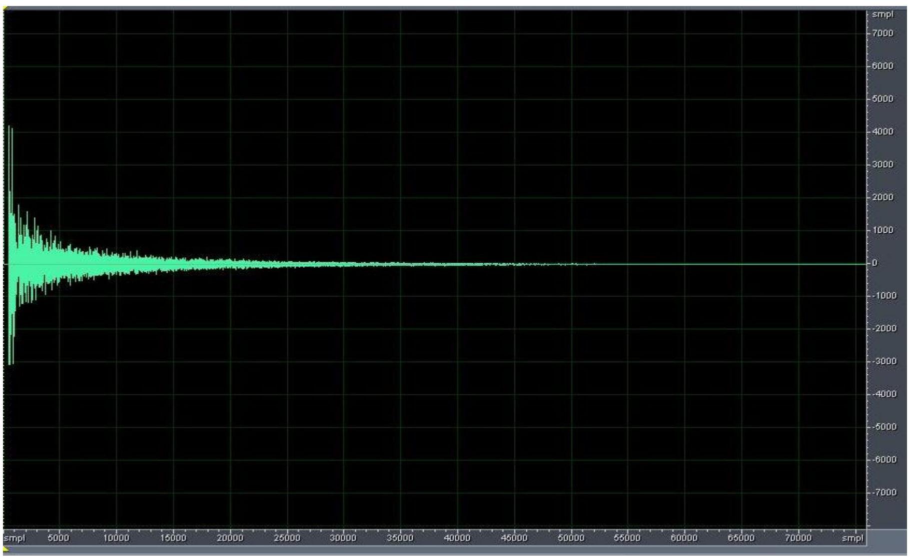
b. Simplify the correcting filter $t(z)$, and set the order of low-pass filter to one.

c. Extract the delay line of left and right channel to make stereo effect under the premise of not increasing calculation amount (One of the good character of FDN).

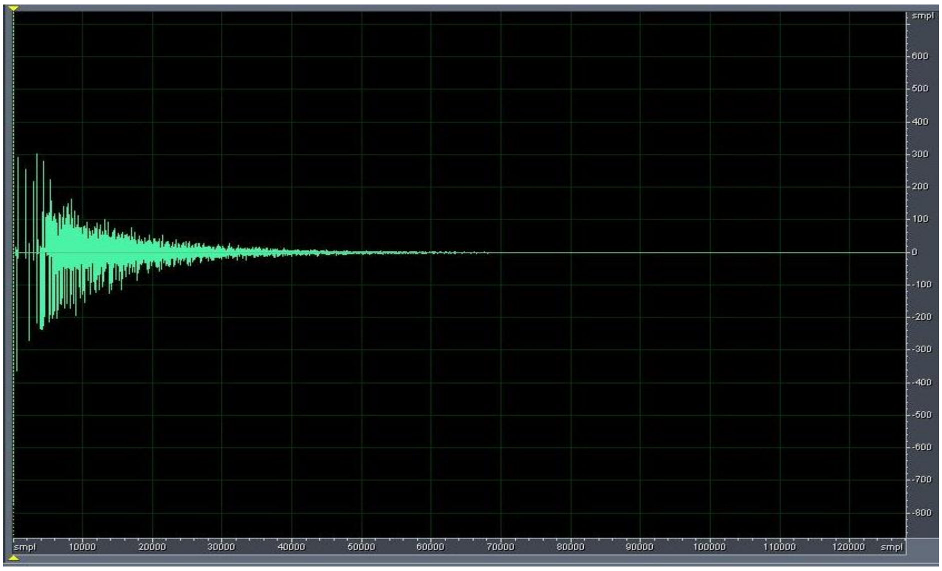
2. Comparisons of envelopes



Impulse Response of MF5602 model (based on traditional algorithm)



Impulse Response of MF5605 model (based on FDN)



World's top reverb plug-in---- TC Native Reverb's pulse response has obvious sign of independent construction of early reflection.

3. Difference between MF5602, MF5605 and TC Native Reverb

a. Apparently, whether TC's model uses FDN as part of its algorithm or not, it deals with early reflection independently of the reverb time. The MF5605 model, same as the original MF5602, does not construct the early reflection independently, therefore it still has some limitations in simulating the natural reverb attenuation process.

b. The FDN algorithm has a conspicuous advantage against the traditional algorithm---- researchers can measure the attenuation in every part of the spectrum of natural reverb through "Energy Decay Relief" technology, and simulate the attenuation process through increasing the order of low-pass filter and the design of Correcting Filter", thus making the digital reverb approaches the natural reverb in the reverb time thoroughly.

V. References

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