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Digital Waveguide Networks for Room Response Modeling and Synthesis

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ABSTRACT

Digital waveguide networks (DWN) are known as a methodology to simulate spatially distributed systems, such as reverberators (room simulation) and resonators of musical instruments. This paper is an overview and study on the application of DWNs to simulate acoustic spaces for room rendering, including auralization. The methods discussed combine the principles of digital waveguide meshes, image source models, reverberation algorithms, and HRTF-based rendering. Examples are given on synthesizing room responses for simple room geometries, and the possibilities of fitting the models to real room responses are discussed. System performance is discussed from the point of view of real-time virtual acoustics.

1. INTRODUCTION

Techniques for simulating acoustic spaces for virtual acoustics can be roughly divided into a few main categories: *geometrical methods* (image source method and ray-based methods), *element- or mesh-based methods* (finite element method, boundary element method, digital waveguide meshes and networks, finite difference meshes), as well as *statistical methods and reverberation algorithms* for late reverberation. These are often combined into hybrid methods in order to utilize the strengths and compensate for the weaknesses of each approach used alone.

The original idea of digital waveguides (DWG) and digital waveguide networks (DWN) was to simulate room reverberation [1]. Yet the main applications of the approach have been in physics-based modeling of musical instruments, and surprisingly little has been published on their use in the original application domain. In this paper we study the use of DWNs to simulate acoustic spaces, including real-time auralization. When comparing the strengths of different room simulation methods, it is evident that the geometrical approaches are attractive in computing the early part of the room response (direct sound and early reflections), while proper modeling of late reverberation that way is very difficult. Image source methods become computationally too expensive for late reflections, while traditional ray-tracing has problems in providing enough temporal density for late responses. On the contrary, reverberation algorithms and statistical models are useful only for the late reverberation.

In principle the element- and mesh-based methods, including multidimensional digital waveguide structures, are physically well motivated for the whole temporal range from the direct sound to late reverberation. The latter property comes from their inherently recursive structure. However, detailed modeling using regular mesh structures is practical only at low frequencies due to excessive computational load for short wavelengths.

Digital waveguide meshes have been applied to 2-D structures such as membranes [2, 3] and 3-D spaces [4, 5, 6]. Advanced mesh grids [7, 8] and interpolation techniques [9, 10] have been developed to counteract the dispersion problems inherent in them. Finite difference meshes, equivalent to basic DWN meshes (see Section 2.4), are used to improve computational efficiency [11] in the modeling of homogeneous fluids such as wave propagation in the air.

While waveguide meshes are regular structures sampling the physical space, *digital waveguide networks* (DWN) [1, 12] have more freedom in topology, and thus they are less rigorous approximations of real spaces. Both digital waveguide meshes and networks have the desirable property that arbitrary structures remain passive and thus stable as far as simple rules of passivity are valid for each element.

The question arises how much the mesh-like structures can be pruned down in complexity, yet retain simulation accuracy good enough for virtual acoustics. Another question of interest in this paper is how DWNs fit to auralization of a simulated room response. This means modeling the head and ears as a simplified mesh or getting directional information of the sound signal at the observation point of head position. Furthermore, for high-quality virtual acoustics the directivity of sound sources should also be modeled.

In this paper we will discuss many of these questions, particularly from the point of view of real-time or maximally efficient simulation of acoustic spaces. The content of the paper is organized as follows. In Section 2 we present an overview of different techniques to simulate acoustic spaces, focusing on the aspects that are important from the point of view of this paper. The design of DWNs for room simulation is studied in Section 3 and auralization aspects are discussed in Section 4. Experiments of real-time simulation of simple rooms is presented in Section 5. Discussion and summary in Section 6 conclude the paper.

2. OVERVIEW OF ACOUSTIC SPACE SIMULATION TECHNIQUES

In this section we present an overview of methods for the simulation of acoustic spaces. The goal is to discuss aspects that are important for understanding the role of digital waveguide networks in the present framework. For a general overview on acoustics modeling and auralization written recently, see for example [13].

The methods of simulating acoustic spaces are generally divided into wave-based and geometry-based approaches. In this section we start from the theoretically most accurate methods, which reflect the wave-based behavior (solving of PDEs, element-based methods, DWGs, and finite difference schemes), then proceed to geometry-based techniques (image source and ray-based methods as well as statistical approaches), and finally discuss reverberation algorithms.

2.1. Solving of PDEs

Wave phenomena, continuous in time and space, are inherently described by *partial differential equations* (PDEs). The fundamental formulation in acoustics is the Helmholtz equation [14]

$$\nabla^2 p + k^2 p = 0 \tag{1}$$

where p is the sound pressure within the space and k is the wave number $k = \omega/c = 2\pi f/c$. Furthermore, f is frequency and c is speed of sound in the air). While analytical solving of the wave equation with initial and boundary conditions is important for conceptual understanding, in practice it is applicable only to idealized special cases. Practical computational techniques always need some discretization to be done. Even if a case could be solved analytically, real-time simulation in practice requires a discrete-time realization of the obtained transfer functions.

2.2. FEM and BEM

Conceptually closest to the traditional analytical methods of solving PDEs are the element-based techniques of *finite element method* (FEM) [15, 16, 17] and *boundary element method* (BEM) [18, 19] modeling, whereby the space needs to be discretized. In practice FEM is applied using specific software where a geometrical model is built as an (irregular) grid of spatial points for the space, parameter values of the medium and surfaces are given as well as sound sources, and a numerical solver is applied. The sound field is solved at frequencies of interest, and temporal responses (impulse responses) are obtained by inverse Fourier transform from the frequency domain data.

The boundary element method is similar to FEM but there the description of the sound field is reduced to representation at boundaries so that the field in any point in the space can then be solved from the boundary-related data.

As FEM and BEM are basically frequency domain techniques, their application to real-time simulation, particularly in cases of time-varying or nonlinear systems, is hardly practical in any case. For off-line computation of acoustic spaces with modern computers FEM/BEM tools[1] are becoming increasingly attractive at low frequencies, typically below 1 kHz, where spaces with complicated geometry are otherwise difficult to model accurately.

An advantage of FEM/BEM is that in the frequency-domain processing used in them fractional delays are not particularly difficult or computationally expensive to deal with, contrary to time-domain simulations.

2.3. Digital waveguides and mesh structures

Digital waveguides [20] are bi-directional delay lines which simulate the propagation of traveling wave variables along one dimension. They are connected to each other via scattering junctions, which redistribute energy entering into a particular junction to all waveguides associated with the same junction (see Fig. 1). The pressure at a junction J at time index n is calculated as:

$$p_J(n) = \frac{2}{\sum_{i=1}^{N} \Gamma_i} \sum_{i=1}^{N} \Gamma_i p_i^+(n), \qquad (2)$$

where N is the number of waveguides connected at the scattering junction, Γ_i is the admittance of waveguide

[1]Examples of FEM/BEM software tools are LMS Virtual.Lab Acoustics, FEMLAB, ANSYS, ABAQUS, MSC/Nastran Acoustics, WASCAT, and Elmer.

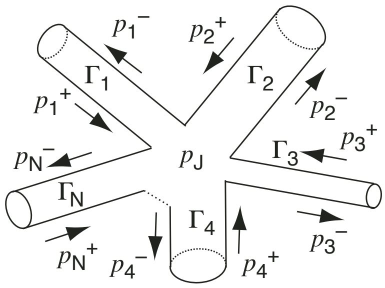

Fig. 1: A scattering junction of connected acoustic tubes. Traveling pressure waves are denoted by '+' for incident and by '-' for scattered wave components.

i, and p_i^+ is the incoming pressure traveling wave to the junction from waveguide i. A scattering junction is energy-preserving — no energy is created or lost, only spread in space and time. The outgoing pressure traveling wave p_i^- from the junction (or alternatively, incoming wave to waveguide i) is calculated at each time step using

$$p_i^-(n) = p_J(n) - p_i^+(n), \qquad (3)$$

which comes from the constraint that pressure at a junction must be continuous, such that $p_J(n) = p_i^+(n) + p_i^-(n)$ for all waveguides i connected at junction J.

A *digital waveguide network* (DWN) [1] is any arrangement of digital waveguides interconnected by scattering junctions. Composed solely of these elements[2], the DWN is lossless and its temporal response increases in echo density over time due to the network structure and the diffusive effects of scattering junctions. For practical applications, losses in the form of gain factors or digital filters must be inserted into the network to produce smooth and exponential decay over all frequencies. DWNs have been used as the late reverberation module of an acoustic environment rendering program [23]. The network topology and waveguide impedances are determined from geometrical analysis of the environment to be simulated based on a path-tracing algorithm.

An acoustics simulation method called mesh-tracing uses a network structure but differs from the DWN algorithm in the way waves propagate through the system [24].

[2]*Wave digital filters* [21, 22] is another wave-based modeling methodology, originally developed for lumped electric circuits. Wave digital filters are compatible with the digital waveguide networks, and they can be used for example to connect lumped elements as loss or dispersion loads to DWN scattering junctions.

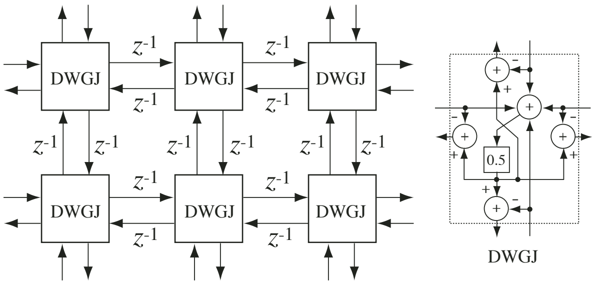

Fig. 2: Left: part of a rectilinear 2-D waveguide mesh structure. Bi-directional unit delays z^{-1} connect scattering junctions (square blocks). Right: computation of a DWG scattering junction.

Nodes are placed according to a uniform random distribution to fill the desired acoustic space and then Delaunay triangulation is applied to determine the connections between the nodes.

When scattering junctions are arranged in a regular grid and connected to neighboring junctions by digital waveguides, the DWN is called a *digital waveguide mesh* [2]. Figure 2 illustrates part of a rectilinear waveguide mesh in two dimensions. From Eq. (2) we can see that for a 2-D mesh with equivalent delay-line admittances the junction pressure is 1/2 times the sum of the incident pressures. For a rectilinear 3-D mesh, which has six waveguides intersecting at each junction, the coefficient is 1/3.

Digital waveguide meshes simulate multi-dimensional wave propagation and have been used to model two-dimensional resonant structures such as percussion membranes [25, 26, 27, 3, 28] and acoustical enclosures such as the vocal tract [29], violin bodies [6], and rooms. In particular, 3-D rectilinear and tetrahedral meshes have been used to simulate the response of a room [4, 30, 31]. Two-dimensional reductions of rooms have also been modeled with 2-D rectilinear and triangular meshes [32, 33]. Frequency-dependent wall and air absorption losses can be simulated by filtering traveling-wave variables at mesh boundaries [34]. In addition, boundaries can be modified for diffuse rather than specular reflection by either explicitly implementing quadratic residue diffusers [35] or by using time-varying circulant matrices to vary the incident angle of traveling waves at the boundaries [36].

Direction-dependent and frequency-dependent dispersion of signal propagation are inherent properties of digital waveguide meshes. In rectilinear meshes, signal propagation is perfect along the main diagonals but along any other direction of travel the wave speed varies with frequency, resulting in high frequency components traveling slower than low frequency components. The direction-dependent dispersion has been counteracted (in FDTD type of waveguide formulations) by spatial interpolation of junction values [9, 10] or by using a mesh grid which has a relatively uniform wave propagation speed along all directions, such as a triangular mesh in 2-D modeling and a tetrahedral mesh in 3-D [7, 8]. However, some dispersion still remains and it is hard to remove without compromising efficiency. The frequency-dependent dispersion characteristics can be corrected by frequency-warping techniques [9, 10], but this is strictly an offline procedure.

From the auditory point of view the effects of frequency-dependent dispersion may even be imperceptible in certain applications since modes have little frequency error at low frequencies and our ears tend to consider only a general modal distribution within each critical band rather than individual modes at high frequencies, where the dispersion error is much greater. So frequency warping may not be needed. The audibility of DWM modeling errors in general remains an interesting topic for future research.

Though digital waveguide meshes are a subset of generalized digital waveguide networks, there are some fundamental differences with regard to room acoustics simulation. While abstract networks often do not have a clear dimensionality or an interpretation in the physical domain, the mesh approach follows a physical modeling paradigm, where the geometry of the modeled room determines the shape and size of the mesh and boundary filters are designed according to absorption and diffusion characteristics of desired surface materials.

This explicit physical description of the acoustical enclosure allows the entire time response to be simulated, from direct sound to early reflections to late reverberation. The distribution of echoes over time and distribution of modes across the frequency range are similar to that in real acoustic spaces, arising naturally from the mesh geometry. The grid variables of the mesh satisfy a discretized wave equation, and as the mesh density goes to infinity, the original wave equation is satisfied. That is, the mesh is a "consistent" finite difference scheme [37].

Auralization is in principle straightforward since each junction in the mesh is associated with a physical location. To obtain a spatialized stereo output signal, it is

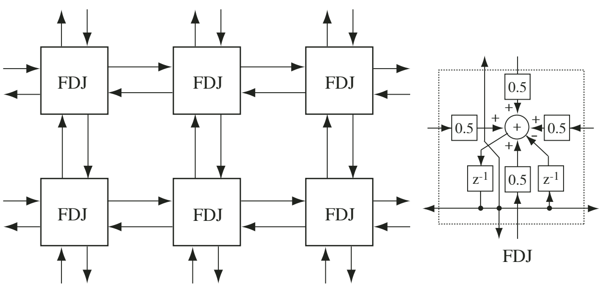

Fig. 3: Left: part of a rectilinear 2-D FDTD mesh structure. Bi-directional delay-free connections link the junctions (square blocks) and the memory z^{-2} is a part of the junction. Right: computation of an FDTD junction.

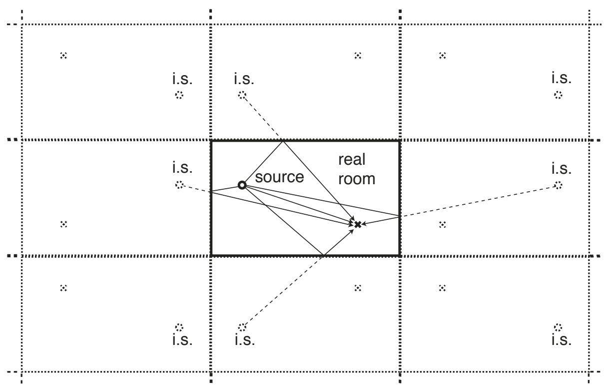

Fig. 4: Source and receiver in a rectangular room (solid line) and their images in the neighboring image rooms (dotted line).

only necessary, in principle, to insert a virtual dummy head having two virtual ears to bring out spatialized sound within the mesh at that location.

Another benefit inherent in the mesh algorithm is that diffraction is automatically simulated by the wave propagation scheme. Unfortunately, the high computational cost of 3-D digital waveguide mesh simulations with full resolution is prohibitive except for modeling small acoustical spaces. Currently it would be practical to use the mesh for low-frequency simulation of rooms and to use a geometric method for simulating the rest of the response over the frequency region where diffraction effects are negligible [4, 30]. Also, more research is taking place for designing digital filters to accurately simulate frequency-dependent absorption [4, 30] and diffusion at the mesh boundaries.

2.4. Finite difference models

While digital waveguides are based on the d'Alembert solution of the wave equation and use traveling wave components for computation, *finite difference time domain* (FDTD) modeling [37, 39, 40, 41, 42] takes another approach. By approximating the differentials of the wave equation by finite differences and selecting the temporal and spatial discretization properly, a mesh structure is obtained. A new junction pressure

$$p_J(n+1) = \frac{2}{\sum_{i=1}^{N} \Gamma_i} \left(\sum_{i=1}^{N} \Gamma_i p_i(n) \right) - p_J(n-1) \quad (4)$$

is calculated by subtracting the previous junction value $p_J(n-1)$ from the weighted sum of N neighboring junction pressures $p_i(n)$.

For a 2-D rectilinear mesh in a homogeneous medium the computation can be organized as shown in Fig. 3. Under some assumptions this is equivalent to the digital waveguide mesh of Fig. 2 [42]. The FDTD mesh has some advantages such as lower memory consumption and faster computation, but is numerically less robust. While the termination (boundary condition) of a mesh is easier to realize using digital waveguide networks, a hybrid of FDTD and digital waveguide meshes can be advantageous [11]. This is made possible by the so-called KW-converter [42] that maps between the wave components and the variables used in the DFTD mesh. Another method for converting between wave components and physical node variables as well as showing the equivalence is described in [43].

2.5. Image source method

The *image source method* (also image method or mirror image method) for computational acoustics draws from the analogy to the behavior of light in a room with reflecting surfaces. It is easy to conceptualize by visual inspection, and for simple room geometries the computation of the image sources and paths from the image rooms (see Fig. 4) is an easy and computationally efficient task. For each path from source to receiver there is need to compute the distance-dependent propagation delay and attenuation of sound, as well as each reflection, which can be combined into a single digital filter

$$H(z) = \frac{A}{r} z^{-r f_s / c} \prod_{i=1}^{N} R_i(z) \quad (5)$$

where source amplitude A is scaled inversely proportional to path distance r, delayed by time rf_s/c due to distance r by speed of sound c scaled to unit samples of sample rate f_s, and filtered in N reflections by $\prod R_i(z)$. Finally the responses of each path are summed together.

The image source method has been used for long time in acoustic room simulation to solve different kinds of numerical problems with different levels of generalizations [44, 45, 46, 47, 48], including the edge diffraction problem [49, 50]. Due to its conceptual simplicity the image source method is used also in real-time virtual acoustics rendering software systems [51, 52] for simulating early reflections. The image source principle is utilized also later in this paper.

2.6. Ray tracing and beam/cone tracing

While the image source method is attractive for the low-order reflections, the finding of possible (visible) paths becomes computationally demanding for the late part of the response. Ray-based methods such as *ray tracing* and *beam/cone tracing* [53, 54, 55, 56] are often combined with the image source method to compute the late response. A large number of rays or beams/cones are tracked from the source point to different directions and the hits to a receiver volume are counted as energy packets with time delay and energy, finally combined into a temporal response. This is a statistical approach—a kind of Monte Carlo simulation. The computation can be done backwards from the receiver to the source as well. Unlike the image method, reflections may be diffuse; that is, each ray incident on a wall or object may give rise to a larger number of reflected rays.

While ray-based methods have been considered as off-line techniques for path-finding and thus not practical for updating paths in real-time rendering of moving sources and receivers, some recent studies show promise to make the beam tracing efficient enough for real-time simulation and auralization [23, 57].

2.7. Statistical modeling techniques

There are methods of room acoustics modeling that can be considered statistical in nature. Even the ray-based methods are statistical in the sense that a high number of rays is needed to approach realism in the response. A somewhat similar method, applied particularly to diffuse spreading phenomena such as light radiation and computer graphics simulation, is the *radiosity method.* For the use of it in acoustics simulation, see for example [58, 13].

Most reverberation algorithms can also be considered as statistical methods. They will be overviewed in the next subsection.

2.8. Reverberation algorithms

Classic artificial reverberation algorithms had the goal of simulating natural-sounding reverberation. These algorithms were developed predominantly from a time domain perspective, with structures designed to imitate typical temporal characteristics in reverberation such as a relatively fast buildup of echoes, high echo density, diffuse sound, and a smooth impulse response envelope. Recursive structures and feedback elements help maintain the recirculation of energy and echoes within the artificial reverberator. Allpass filters expand individual echoes into multiple echoes and scattering matrices redistribute energy within the reverberator, creating a diffusive effect. Gains and lowpass filters placed within the reverberator control the reverberation time and provide frequency-dependent exponential decay. An excellent source on artificial reverberation techniques is [59].

The reverberation algorithms discussed in this section refer to the late reverberation module of a complete reverberation algorithm. The early reflections are usually handled separately for greater realism and accuracy, using a sparse FIR filter or a delay line with filtered taps to process the source sound [60]. A geometric method such as ray-tracing or the image source method is used to calculate the low-order reflections. The arrival times, spherical spreading attenuation, and instances of reflection calculated from the geometrical analysis determine the sparse FIR filter coefficients or the tap locations, gains, and associated filters for the delay line. Depending on the late reverberation algorithm, either the direct sound alone or the entire set of early reflections are fed into the late reverberation module. The latter option helps to build up echo density more quickly in the reverberator.

Early reverberators were based on combinations of feedback comb filters and allpass filters, (see Fig. 5). One of the earliest was a structure by Schroeder with four comb filters in parallel feeding into two allpass filters in series [61]. This was improved by Moorer, who introduced a lowpass filter into the feedback loop of the comb filter to generate a more natural-sounding decay. His preferred structure was a parallel bank of six lowpass comb filters which fed into a single allpass filter [62]. A comb filter, whose resonances are harmonic, can be considered as modeling a plane wave trajectory whose path length

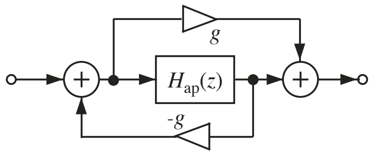

Fig. 5: Generic filter structure for building reverberators. When $H_{\mathrm{ap}}(z) = z^{-N}$, i.e., a delay of N unit samples, this is an allpass filter $H(z) = (g + z^{-N})/(1 + gz^{-N})$, including the first-order case $N = 1$. When $H_{\mathrm{ap}}(z)$ itself is an arbitrary allpass filter, the structure becomes a nested allpass filter. When the forward path (g) is removed, it becomes a feedback comb filter, and when the feedback ($-g$) is removed, it becomes a feedforward comb filter.

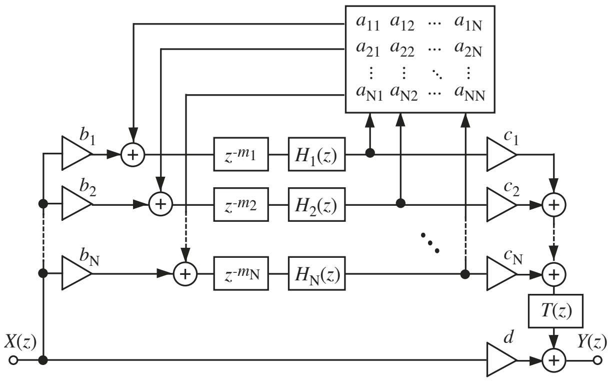

Fig. 6: The structure of a feedback delay network (FDN) consisting of a unitary matrix $\mathbf{A}$, delays z^{-m_i}, lowpass filters $H_i(z)$, coefficients b_i, c_i, and d, and compensation filter $T(z)$.

corresponds to the comb filter length. An allpass filter serves to increase the echo density and to make the impulse response more diffuse.

A more sophisticated structure was developed by Gardner which had a cascade of allpass filters, delay lines, and nested allpass filters [63]. The output of the cascade was attenuated, filtered, and then fed back to the cascade input. A nested allpass filter is a reverb unit which, unlike the regular allpass filter, enables echo density to build up over time. Dattorro's reverberation network [64] for simulation of a plate reverberator first processes the input through a lowpass filter and cascade of four allpass filters in order to diffuse and decorrelate the signal. The signal then enters a recirculating "tank" which features allpass filters with time-varying delays. These help diffuse the signal even further and eliminate patterns within the tank.

A different approach was taken by Kendall, whose spatial reverberator used recirculating delays to implement an image source model [65]. Two different reverberation units resembling lowpass comb filters were designed to simulate first-order reflections (coming from virtual rooms parallel to the real room) and second-order reflections (coming from virtual rooms diagonal to the real room). These units were run in parallel and also crossfed in order to approximate reflections from virtual rooms in between those on the axial and diagonal axes. The crossfeeding also increases the echo density over time. Though the parameters for the reverberation units are easily computed from the geometric analysis, the algorithm is not able to support complex room shapes.

The *feedback delay network* (FDN) was first introduced by Gerzon [66, 67] but also presented independently by Stautner and Puckette about a decade later [68]. Jot and Chaigne [69] developed the FDN model further, into its most common representation as shown in Fig. 6. The FDN consists of a number of delay lines in cascade with (lowpass) filters whose outputs enter a unitary scattering or mixing matrix. The outputs of the scattering matrix in turn feed back into each of the delay lines. The coefficients of the mixing matrix can be chosen to control the amount of diffusion (e.g., a diagonal matrix results in parallel comb filters) and the degree of correlation (e.g., an orthogonal matrix decorrelates the signals) in the system. A modification of the FDN to include an allpass filter in series with each delay line and lowpass filter cascade results in a faster buildup of reflection density and may lower the FDN order needed to produce good reverberation [70].

The digital waveguide network (DWN), described earlier in Section 2.3, was proposed by Smith for simulating late reverberation [1]. Its strengths and potential as a reverberator are similar to those of the FDN: it can be formed as a lossless, stable prototype so that the reverberator structure and the frequency-dependent losses can be handled and analyzed separately; it is a recursive structure with scattering functions which allows echo density to increase over time as well as the response to become more diffuse; and it is often sparse and efficient. FDNs can be interpreted as special cases of DWNs and can be embedded within larger DWNs [71, 59]. The many similarities of the two paradigms are made visible when they are both expressed as sparse state-space models (state-space models using delay-lines in place of unit delays).

Other reverberation algorithms of note include convolution [72] and a multirate reverberation system [73]. Convolution is computationally intensive and rather inflexible in producing different types of reverberation. However, a hybrid convolution algorithm has been developed which does not have audioble input-output delay [74]. The multirate algorithm measures and approximates a room impulse response by obtaining the sub-band impulse responses in parametric form. However, this method is not suitable for real-time applications [75]. More detailed information on reverberation algorithms and multi-channel reverberation rendering can be found in [60, 72, 75].

Reverb design

A general difficulty in designing artificial reverberators is that very often there is no prescriptive formula beyond avoiding delay lines lengths with common factors, which cause overlapping of echoes and modes. The design process usually relies on a good ear, intuition, and much trial and error. Many of the parameters and structures are determined empirically, such as Gardner's and Dattorro's reverberators, and it is often difficult to perform consistently well over a wide parametric range (e.g., a good small room reverberator may not scale well to simulate reverberation in a large room, or a certain algorithm sounds poor with a long reverberation time while sounding decent with a short decay).

Some algorithms, such as the parallel comb filters, have basic design guidelines [75]. A minimum time density constraint — at least 10000 echoes per second [76] — determines how many comb filters are needed in parallel. A minimum modal density constraint — at least 0.15 eigenfrequencies per Hz [61] — affects the choice of delay lengths and puts a lower limit on the total length of all the delays.

For algorithms that have a physical interpretation, possibilities open up for creating a wide and controllable range of impulse responses which have perceptual similarities to actual acoustic spaces. In a FDN, the delay line lengths may be set to correspond to room resonances [77]. Aside from Sarti and Tubaro's digital waveguide network implementation based on beam-tracing analysis [23], a DWN has two other relatively straightforward design possibilities. In the form of a digital waveguide mesh, a DWN can be used to explicitly model the geometry and acoustical features of a room. Under certain conditions a DWN can be equivalent to a FDN [71], and therefore its waveguide lengths may be set according to the FDN delay line lengths mentioned earlier. In Section 3 we will discuss the principles of designing relatively simple DWNs that model both the early reflections and the late reverberation.

3. DIGITAL WAVEGUIDE MODELING AND SYNTHESIS OF ROOMS

In this section we discuss the principles of applying DWNs to room acoustics modeling and simulation, particularly from a real-time auralization point of view. First we discuss the possibility to prune down the density of a regular mesh structure. The main part of the analysis is on simplified digital waveguide networks consisting of a fairly small number of bi-directional delay lines and scattering nodes connecting them. In practical implementations filters are also needed within delay lines or scattering junction nodes to control the losses and dispersion of wave propagation. We are interested in DWN structures that do as much of the room simulation as possible, not just late reverberation combined with a separate early reflection simulation, by other means. Receiver design for auralization is discussed in Section 4.

3.1. Reducing the density of meshes

Mesh-based models require in practice a spatial density of about 6 mesh points per wavelength to be accurate enough by physical criteria [5]. For an audio bandwidth of 10 kHz this means a mesh point distance of approximately 0.5 cm. For a medium room size of 4 x 6 x 3 m^3 this means 800 x 1200 x 600 = 576000000 mesh points, and for a concert hall of 30 x 60 x 10 m^3 this means 144000000000 mesh points. While the former one requires "only" a few gigawords of memory, the time consumed for simulation is far beyond real time even in the forseeable future, and for the concert hall hopelessly impractical even as a non-realtime simulation.

If the bandwidth of simulation is reduced, the mesh size required decreases rapidly, being proportional to f_s^3 in 3-D modeling, where f_s is the sampling rate. Another choice is to reduce only the mesh node density by using delay elements of multiple unit delays, which is still computationally efficient when they are realized as circular buffers. Then there will be no frequency aliasing at the full sample rate, only spatial aliasing that leads to smearing of the direction of arrival as well as ripples in the magnitude response. The reduced-density mesh is an interesting case from the viewpoint of the human auditory system, which is sensitive to interaural time difference (ITD) only up to about 1.5 kHz and above that primarily sensitive to interaural level difference (ILD) only.

Another helpful detail could be the use of higher mesh density around the source and the receiver and lower density elsewhere. Particularly when a 2-D model without elevation cues is enough for auralization, the dimensionality of modeling comes down close to practical simulations.

A problem with reduced mesh density, in addition to decreased spatial accuracy, is that the dispersion inherent in 3-D and 2-D mesh-based modeling gets worse, although this may not be a perceptually important problem. The density pruning principles are not discussed further in this paper.

3.2. Realization of losses and dispersion in DWN room models

Dispersion and diffuseness[3] in artificial reverberation is obtained through the use of allpass and comb filter structures, as discussed in subsection 2.8. In physical rooms these acoustic effects as well as losses take place typically in reflections and scattering at boundaries, including edge diffractions. Therefore, it is interesting to investigate DWN cases where scattering junctions at wall positions are designed to implement dispersive and lossy reflections to DWN delay lines by approximating physical reflections. The *reflectance* $S(z)$ is defined here as the ratio of reflected (-) and incident (+) pressure signals

$$S(z) = P^{-}(z)/P^{+}(z). \tag{6}$$

If the wave admittance of a delay line attached to the junction is $\Gamma_{\mathrm{R}}(z)$, then a given pressure wave reflectance $S(z)$ can be realized by loading the junction by admittance $\Gamma(z)$ so that

$$S(z) = \frac{\Gamma_{\mathrm{R}}(z) - \Gamma(z)}{\Gamma_{\mathrm{R}}(z) + \Gamma(z)} \;\leftrightarrow\; \Gamma(z) = \frac{1 - S(z)}{1 + S(z)}\,\Gamma_{\mathrm{R}}. \tag{7}$$

[3]Dispersion means here the temporal spreading of frequency components (allpass transfer function with frequency-dependent delay) of reflections or wave propagation. That is what can be done with filters in the DWN structures. Diffusion refers to the spreading of wavefront directions at boundary reflections, which does not take place in simple DWNs due the small number of possible wave propagation directions. For an overview of diffusion modeling, see [78].

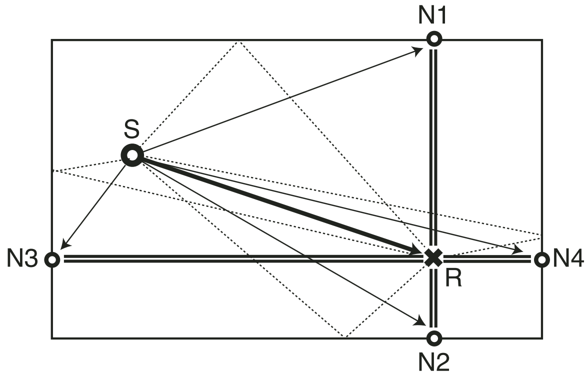

Fig. 7: Simple DWN configuration in a rectangular room in the horizontal plane. S = sound source, R = receiver, and N1-N4 are scattering junctions. Thick arrow line: direct sound path; thin dotted lines: real acoustic paths of first-order reflections; and thick double axial lines: waveguides (bi-directional delay lines). The thin solid arrow lines are for feeding junctions to approximate the first-order reflection paths.

For example, a measured or synthesized reflectance can be used by loading a reflection junction by a proper wave digital admittance [21] or by realizing scattering coefficients according to Equations (2) and (3).

3.3. Simple DWN for rectangular rooms

To gain intuition about the possibilities of WDN modeling we will start from simple cases with a sound source and a receiver in arbitrary positions in a rectangular room. The conceptual framework for the early sound is based on the image source modeling approach. For simplicity, only the horizontal plane is considered. Extending to the vertical dimension is conceptually straightforward.

Figure 7 shows a sound source (S) and a receiver (R) along with the direct sound path (thick arrow line S-R) and the first-order reflection paths (thin dotted lines). A highly simplified 2-D waveguide network consists of two waveguides (double-lines N1-N2 and N3-N4) through the receiver point and perpendicular to the walls of the room. This configuration is geometrically simple for real-time auralization and dynamic control of model parameters for moving sources and receivers.

It is easy to notice that the direct sound always needs special treatment, i.e., just a delay from S to R with a proper gain factor for attenuation. This path is totally independent of the walls and size of the room so it cannot utilize

DWN nodes that are on the walls, which is a requirement to realize the temporal structure of reflections between walls.

Let's study first a simplistic case where the waveguides N1-N2 and N3-N4 are uncoupled. Both waveguides are simple bi-directional delay lines with terminations at the walls, so they can be designed independently for frequency-dependent decay and dispersion by adding proper lowpass filtering and allpass structures for dispersion inside each delay line loop. The waveguides also need to be fed by delayed and scaled excitation inputs from the source S (thin arrow lines) to nodes N1-N4. The receiver R just senses the incident wave components in the delay lines for auralization.

The inherent limitations of simple DWNs become evident from this configuration. While the direct sound is treated separately and is therefore no problem, already the first-order reflections arrive from incorrect directions (the arrival times can be adjusted by the delays from source S to nodes N1-N4). Another general problem in addition to the direction error is the signal level error of reflections. This can be noticed for example when the source is near any of the nodes, say N2. For the first reflection the source coupling to that node should be strong, but this leads to the fact that the later reflections along that waveguide N1-N2 will remain almost as strong (unless decay time is very short). This problem comes from the spherical nature of the wavefront near the source, which doesn't fit naturally to the sparse DWN structure.

The next limitation of the simplistic case in Fig. 7 is that wave components and thus also modes are possible only axially to the wall directions, and no cross-modes can emerge. This may be even desirable for special room effects, such as strong flutter echos, but not for yielding perceptually valid rendering of normal room acoustics. By adding enough dispersive elements within the simple DWN model, however, it can be made a generic room simulator without noticeable artifacts.

3.4. Improving the simple DWN room model

One possible improvement to the simple model is shown in Fig. 8. The waveguides marked by dual-dotted lines between nodes N1-N4 have been added to allow for coupling of the axial waveguides[4] and to allow the build-up of cross-modes, although the receiver still perceives reflections as coming only from axial directions.

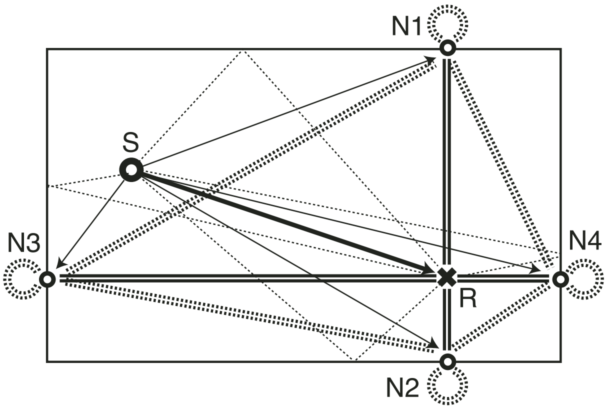

Fig. 8: DWN configuration in a rectangular room with added non-axial waveguides (dotted double lines) and loading of nodes for losses and dispersion/diffusion (circular dotted double lines).

Another minor detail is the possibility to move the nodal points N1-N4 at the walls so that the first-order reflections come from directions closer to the arrival angles in the real room. However, this means that all later reflections also come from these directions. If the accuracy of first-order reflections is crucial, they can be realized in a way similar to the direct sound, i.e., by separate delays from the source. This kind of special treatment of early reflections means, however, that the DWN structure is used more or less for late reverberation only.

Compared to the case of Fig. 7, the waveguides in Fig. 8 are truly connected. Thus the design for losses and dispersion/diffusion is more complex. From a physical point of view, most of these phenomena take place in reflections at the walls. Thus it is natural to keep the waveguides lossless and dispersionless (except for minor air absorption loss), and simulate the phenomena in the nodes at the walls. Losses can be added by loading the junctions by wave digital admittances [21, 22] that have a proper resistive component for absorption simulation. Dispersion/diffusion can be introduced by reactive loading. This can be realized with different recrsive structures, such as star-like DWNs [71], wave digital admittances as discussed in subsection 3.2, or circularly connected waveguides as characterized in Fig. 8.

[4] Another way to achieve coupling between the axial waveguides is to make a scattering junction at the receiver position R with weak scattering coefficients between the main directions. This is, however, not motivated from real physical behavior of rooms.

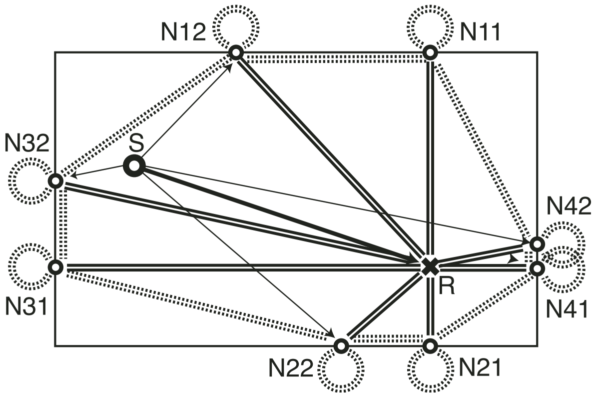

Fig. 9: A DWN structure where secondary waveguides (N12-R-N22, N32-R-N42) are added to correct arrival directions of first-order reflections. For improved cross-modes and reflections, all nodes are connected into a polygon path (dotted double lines). Losses and dispersion/diffusion at the walls is increased by circular connections at each node (circular dotted double lines).

3.5. Adding nodes to DWN models

DWN room models can be improved by adding nodes and delay lines between them. There is an unlimited number of different configurations between the simple models discussed above and a full-density digital waveguide mesh. Here we briefly discuss another relatively simple DWN structure.

Figure 9 depicts one of the many possible but not yet highly complex structures. Secondary waveguides (N12-R-N22 and N32-R-N42) have been added which allow correct first-order reflection directions and improve directional diversity of wave arrivals. Notice that these lines make an angle at the receiver point and thus do not simulate physical wavefront behavior. The DWN nodes at the walls are connected into a polygon structure of waveguides (dotted double lines) to improve the cross-mode behavior. Each node is also circulated to itself through a dispersive structure as was already shown in Fig. 8.

There are obviously infinitely many DWN topologies that could be applied to approximate source-receiver paths in room acoustics. One possibility to extend the models discussed above is to add scattering nodes within the room, not only at the walls. Although its physical motivation is not clear, it may lead to perceptually useful DWNs. In general any addition of nodes and waveguides increases complexity in many ways: complexity of design, complexity of control (runtime computation of parameters), and computational complexity of the signal processing itself within the DWN structure. Although the computation of a single delay, loss and dispersion filter, or a scattering junction is straightforward and fairly simple, the overall complexity grows rapidly. Thus it is desirable to limit the number of DWN elements and to optimize the behavior according to perceptual criteria, whereby the direct sound should be most accurate, first reflections relatively accurate, later reflections just to support overall impression. Finally the late reverberation has to be diffuse enough without audible artifacts.

3.6. Approximation of complex room shapes

If the geometry of the room to be simulated is more complex or there are reflecting and diffracting objects inside the room, the image source approach for approximating the waveguide junction positions is still a generally useful guideline. Diffraction from edges can also be taken into account. In fact, edge diffraction modeling and image source modeling can be integrated together [49, 79]. If the first order (and second order) reflections and diffractions need accurate directional and level estimates, it is probably better to implement them as separate image source paths than to try to realize them as part of a DWN structure.

3.7. Good late reverberation

Since the times of Schroeder [61] and Moorer [62] the goal of artificial reverberation has been naturalness without artifacts, but typically in a generic form and not calibrated in detail to any specific room. The complexity of reverberation in real spaces is high [80] and the auditory system is not very sensitive to fine details. Therefore modeling the early part with more accuracy by various techniques and combining this to generic late reverberation works fine in many cases.

Important features of generic late reverberation are a lack of undesirable spectral peaks or valleys and a response envelope without periodicities in in any auditory critical band. The quality and artifacts can be evaluated to some degree by auditory modeling analysis [81, 82]. In the future this could also be a way to automatically adjust room model parameters to fit perceptually to given (measured) room responses.

In DWN room simulation discussed in this paper the late reverberation is closely integrated with the early response. Therefore there is need to compromise between these requirements. Because the topology of DWNs is in large part dictated by room geometry and positions of the source and the receiver, the temporal and spectral richness of reverberation can be effected mainly by dispersive elements in the model.

4. AURALIZATION OF DWN ROOM MODELS

For virtual acoustics applications, the result of room simulation needs to be rendered, i.e., auralized by loudspeaker or headphone reproduction [83, 84, 51, 85]. For loudspeaker reproduction there are two basic cases: various multichannel techniques or the binaural (transaural) technique. In binaural reproduction by headphones or loudspeakers the goal is to get proper signals into the listener's ear canals. In multichannel loudspeaker techniques the goal is to reproduce the desired wave field at the position of the listener's head. Here we discuss separately two cases: utilizing the wave components and simulation of the sound pressure field.

4.1. Auralization of wave components

When the wave components arriving at the receiver position are computed, including the wavefront direction, the auralization can be done by panning techniques or HRTF (head related transfer function) processing. Amplitude panning, particularly the *vector base amplitude panning* (VBAP) [86], is a straightforward and computationally efficient method to create virtual sound sources in desired directions both in 3-D and 2-D (horizontal) reproduction. *Ambisonics* [87] is another method where a similar approach can be applied. Referring to the figures in Section 3, for each delay line entering at the receiver point, the corresponding signal needs to be panned to the direction of signal arrival.

A problem with loudspeaker reproduction is that the listener cannot move in a space without leaving the sweet-spot area. For a single subject, listener tracking can be done and the varying position can be taken into account for example in the VBAP method. However, for multiple listeners or for cases where the subject comes close to a single loudspeaker, the method may not work well. In such applications *wave field synthesis* [88] is a general solution, but at the cost of a high number of loudspeakers and large amount of related signal processing.

For headphones and binaural loudspeaker reproduction the directions of arrival can be realized by HRTF processing. For directional accuracy the first wave front is the most important one, and should therefore be processed with highest precision HRTFs. Due to the precedence effect the early reflections contribute mostly to timbre and perceived source width, and practically not at all to perceived source direction. Thus the accuracy of HRTFs for those paths can be compromised to some degree.

To make HRTF-based auralization applicable to subjects moving in a virtual space, head-tracking has to be applied and the signal paths need to be updated accordingly.

4.2. DWN modeling of the head

One possibility for headphone auralization, particularly with digital waveguide mesh structures, is to model the head as a mesh as well. This is like simulating virtual ears on the virtual dummy head. The head can be assumed to be a hard object, spherical or more detailed in shape. To obtain a detailed HRTF behavior a highly detailed DWN model is needed, which means that real-time processing is probably not possible.

An interesting question is if sparse DWNs could be used so that the nodal density is increased for the head area only. The field could be simulated at the ear canal entrance positions or on a circle around the head. In the latter spherical case the *motion tracked binaural* method (MTB) [89] could be applied for head rotation without need to update the mesh itself.

The problem with sparse DWNs that include head modeling is that there is no simple way to design arbitrary networks to obtain realistic propagation and scattering of sound around the head. Thus these methods remain a challenge for future work.

5. DWN MODELING EXPERIMENTS

The principles of simplified DWN room models discussed in Section 3 were simulated in BlockCompiler [90], a block-based modeling environment for real-time simulation and synthesis. BlockCompiler supports multiple modeling paradigms including digital waveguides, wave digital filters, finite difference schemes, hybrid models of these, as well as regular block-based DSP computation. Simple graphical user interfaces were applied to visualize the room geometry and to move the source and the receiver by mouse during real-time simulation.

The goal was to gain basic understanding of the possibilities to integrate early and late parts of room response through the use of geometrically intuitive DWN models. Simple binaural headphone auralization was applied to listen to signals such as speech and music when processed through the models. Here we present some basic experiences from the simulations.

Computational effciency

Sparse digital waveguide networks are computationally efficient. The basic models of Section 3 take about 5-15% of CPU time on a 1 GHz G4 PowerPC processor when a single source and a single receiver are simulated and no specific optimization of the software implementation is done. Thus it is possible to add much to the model complexity if the quality and details of sound are important, especially when complex dispersion models are needed.

Adding new sources increases the computation load less than by the number of sources, while each receiver needs in most cases duplicating the DWN structure. Multiple sources and receivers definitely expands the complexity quite rapidly. In this sense these DWNs are computationally more expensive than algorithms where the DWN principle is used only for reverberation that is common to any receiver in the room.

Simulation of true spatial distribution in the models of Section 3, though in a simplified manner, cannot fully compete in efficiency with source-filter type of models, such as FDNs, or even with simple DWNs, such as the star-like DWN reverberator [71]. This is due to the overhead of using bi-directional dual delay lines and non-vectorizable data structures. In this sense it is hard to compete with the FDN type of simulation by matrix operations. However, for a single-source single-receiver case the difference in computational load is not radical, and the speed of modern computers helps using complex DWN topologies. An advantage of the DWN models discussed in Section 3 is the direct connection of model parameters to the geometry of the room to be modeled.

Modeling of early reflections

As discussed in Section 3, modeling of the early (especially the first-order) reflections is problematic, particularly when the source is close to a scattering node at a wall. By keeping the coupling from the source to such a node below a reasonable level the models work fairly well, because the source itself radiates in such cases from the direction of the node anyhow (see Figs. 7-9).

For high quality early reflections there is probably only one way to go, that is, to realize them separately as delay lines form source to receiver according to the image source principle. This does not increase radically the computational load unless there are many reflecting surfaces or complex dispersion filters in each path. Even when realizing early reflections separately, it is important to excite the DWN structure as early as possible for the build-up of reflections and modes.

Tuning and control of model parameters

The selection of model parameters comes partly from the room geometry, partly it can be derived from measurements of a real room, but quite much it is art of guessing and perceptual tuning, as with any approximate modeling of rooms. The lengths of delay lines is an easy case, determined by the geometry of the room and positions of the source and the receiver. In our experiments the delay lines were lossless and dispersionless, based on the idea that in small and medium-sized rooms the wave propagation in the air behaves that way. If the first- or second-order reflections are implemented separately, they need filters to simulate frequency-dependent effects in these paths.

The coupling factors of the source to the scattering nodes through delays is already a question where good compromise rules are needed, due to the proximity problem discussed before. Rules might be derived for example for the amplitude balance of subsequent reflections, or auditory perception may be used as a guide to find such rules.

Losses and reverberation time are fairly easy to tune in the simplest model of Fig. 7, because each waveguide branch is independent. (This could be achieved also in the model of Fig. 9 if the polygonal path through the scattering nodes is omitted.) Then the reflections at the walls can be designed to realize losses and dispersion. In more complex configurations the control of reverberation time as a function of frequency becomes less easy.

Designing good dispersion structures at the scattering nodes (or as part of waveguide delay lines) is one of the most important and difficult factors to make the models sound good and realistic. This is where good guesswork and hard iteration has given the most natural sounding results in reverb design. An interesting challenge in the room-related DWNs is to utilize measured data from a given room. In the model of Fig. 7 the reflectance at the scattering node could be made to correspond to an average measured reflectance of the corresponding real surface. In cases of more complex DWN topologies discussed above the design of scattering node dispersion becomes also more involved. As with good reverb algorithms, time variance of parameters, such as delay line lengths, can help to remove audible artifacts. A further rule for good dispersion design is to avoid perfect symmetries by making opposite wall dispersions a bit different, unless the goal is to simulate flutter echos and special effects.

Perceptual observations

When listening to the DWN room models described above through real-time headphone auralization it was found that it is fairly easy to make realistic sounding simulation for small to medium sized rooms having relatively short reverberation times. For large spaces and long reverberation times the design of dispersion becomes more demanding, and the simple DWN topologies may not model details of more complex geometries, at least unless more of the early reflections are implemented separately according to the image source principle.

So far the model parameters in the experiments have been adjusted more or less by heuristics and perceptual iteration. Formal listening tests are needed to evaluate the modeling and auralization results, comparing them both to real spaces and other modeling methods. More systematic approaches are necessary to optimally calibrate the model parameters, both by acoustic and perceptual criteria. One of the goals for future research is also to develop computational models based on auditory perception to predict the quality of a model compared to a real room.

6. SUMMARY

This paper has first presented an overview of methods for room acoustics simulation, particularly for real-time applications with auralization, and then concentrated on the use of digital waveguide networks for such purposes. The methods discussed combine the principles of digital waveguide meshes, image source models, reverberation algorithms, and HRTF-based rendering. Examples were given on synthesizing room responses for simple geometries, and possibilities of fitting the models to real room responses were discussed.

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8. REFERENCES

[1] J. O. Smith, "A new approach to digital reverberation using closed waveguide networks," in *Proc. 1985 Int. Computer Music Conf.*, (Vancouver, Canada), pp. 47–53, 1985.

[2] S. A. Van Duyne and J. O. Smith III, "Physical modeling with the 2-D digital waveguide mesh," in *Proceedings of the International Computer Music Conference*, (Tokyo, Japan), pp. 40–47, Sept. 1993.

[3] F. Fontana and D. Rocchesso, "Physical modeling of membranes for percussive instruments," *Acustica United with Acta Acustica*, vol. 84, pp. 529–542, May/June 1998.

[4] L. Savioja, T. Rinne, and T. Takala, "Simulation of room acoustics with a 3-D finite-difference mesh," in *Proceedings of the International Computer Music Conference*, (Århus, Denmark), pp. 463–466, Sept. 1994.

[5] L. Savioja, *Modeling Techniques for Virtual Acoustics*. Helsinki University of Technology, 1999. Doctoral Thesis, Report TML-A3.

[6] P. Huang, S. Serafin, and J. O. Smith III, "A waveguide mesh model of high-frequency violin body resonances," in *Proceedings of the International Computer Music Conference*, (Berlin, Germany), Sept. 2000.

[7] S. Van Duyne and J. O. Smith, "The tetrahedral digital waveguide mesh," in *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, (New Paltz, NY), pp. 463–466, 1995.

[8] F. Fontana and D. Rocchesso, "Signal-theoretic characterization of waveguide mesh geometries for models of two-dimensional wave propagation in elastic media," *IEEE Transactions on Speech and Audio Processing*, vol. 9, pp. 152–161, Feb. 2001.

[9] L. Savioja and V. Välimäki, "Reducing the dispersion error in the digital waveguide mesh using

interpolation and frequency-warping techniques," *IEEE Transactions on Speech and Audio Processing*, vol. 8, pp. 184–194, Mar. 2000.

[10] L. Savioja and V. Välimäki, "Interpolated rectangular 3-D digital waveguide mesh algorithms with frequency warping," *IEEE Trans. Speech and Audio Processing*, vol. 11, no. 6, pp. 783–790, 2003.

[11] M. Beeson and D. Murphy, "Roomweaver: A digital waveguide mesh based room acoustics research tool," in *Proc. 7th Int. Conf. on Digital Audio Effects*, (Naples, Italy), pp. 268–273, 2004.

[12] D. Rocchesso and J. O. Smith, "Generalized digital waveguide networks," *IEEE Trans. Speech and Audio Processing*, vol. 11, no. 3, pp. 242–254, 2003.

[13] U. P. Svensson, "Computational modelling and simulation of acoustic spaces," in *Proc. 22nd Int. AES Conf.*, (Espoo, Finland), pp. 1–20, 2002.

[14] P. M. Morse and K. U. Ingard, *Theoretical Acoustics*. Princeton University Press, USA, 1965.

[15] O. C. Zienkiewicz and K. Morgan, *Finite Elements and Approximation*. John Wiley and Sons, New York, 1983.

[16] O. C. Zienkiewicz and R. L. Taylor, *The Finite Element Method Vol. 1: Basic formulation and linear problems, 4. ed.* McGraw-Hill, London, 1989.

[17] O. C. Zienkiewicz and R. L. Taylor, *The Finite Element Method Vol. 2: Solid and fluid mechanics, dynamics and non-linearity, 4. ed.* McGraw-Hill, London, 1991.

[18] C. A. Brebbia, J. C. F. Telles, and L. C. Wrobel, *Boundary Element Techniques: Theory and Applications in Engineering*. Springer Verlag, Berlin, 1984.

[19] C. A. Brebbia and R. D. Ciskowski (ed.), *Boundary Element Methods in Acoustics*. Computational Mechanics Publications, Southampton, 1991.

[20] J. O. Smith, "Physical modeling using digital waveguides," *Computer Music J.*, vol. 16, no. 4, pp. 74–87, 1992.

[21] A. Fettweis, "Wave digital filters: Theory and practice," *Proc. IEEE*, vol. 74, pp. 270–327, Feb. 1986.

[22] S. D. Bilbao, *Wave and Scattering Methods for the Numerical Integration of Partial Differential Equations*. PhD Thesis, Stanford University, May, 2001.

[23] A. Sarti and S. Tubaro, "Low-cost geometry-based acoustic rendering," in *Proceedings of the COST G-6 Conference on Digital Audio Effects*, (Limerick, Ireland), pp. 19–22, Dec. 2001.

[24] B. Schiettecatte, A. Nackaerts, and B. D. Moor, "Real-time acoustics simulation using mesh-tracing," in *Proc. Int. Computer Music Conf.*, (Singapore), pp. 83–86, 2003.

[25] J. Laird, P. Masri, and N. Canagarajah, "Efficient and accurate synthesis of circular membranes using digital waveguides," in *Proceedings of the IEE Colloquium on Audio and Music Technology*, (London), pp. 12/1–12/6, Nov. 1998.

[26] M. Aird, J. Laird, and J. ffitch, "Modelling a drum by interfacing 2-D and 3-D waveguide meshes," in *Proceedings of the International Computer Music Conference*, (Berlin, Germany), pp. 82–85, Sept. 2000.

[27] F. Fontana and D. Rocchesso, "A new formulation of the 2D-waveguide mesh for percussion instruments," in *Proceedings of the XI Colloquiun on Musical Informatics*, (Bologna, Italy), pp. 27–30, Nov. 1995.

[28] S. A. Van Duyne, J. R. Pierce, and J. O. Smith III, "Traveling wave implementation of a lossless mode-coupling filter and the wave digital hammer," in *Proceedings of the International Computer Music Conference*, (Århus, Denmark), pp. 411–418, Sept. 1994.

[29] J. Mullen, D. Howard, and D. Murphy, "Acoustical simulations of the human vocal tract using the 1D and 2D digital wavegide software model," in *Proceedings of the COST G-6 Conference on Digital Audio Effects*, (Naples, Italy), pp. 311–314, Oct. 2004.

[30] L. Savioja, J. Backman, A. Järvinen, and T. Takala, "Waveguide mesh method for low-frequency simulation of room acoustics," in *Proceedings of the International Congress on Acoustics*, vol. 2, (Trondheim, Norway), pp. 637–640, June 1995.

[31] G. Campos and D. Howard, "A parallel 3D digital waveguide mesh model with tetrahedral topology for room acoustic simulation," in *Proceedings of the COST G-6 Conference on Digital Audio Effects (DAFx)*, (Verona, Italy), pp. 73–78, Dec. 2000.

[32] D. T. Murphy and D. M. Howard, "2-D digital waveguide mesh topologies in room acoustics modelling," in *Proceedings of the COST G-6 Conference on Digital Audio Effects (DAFx)*, (Verona, Italy), pp. 211–216, Dec. 2000.

[33] D. T. Murphy, C. J. C. Newton, and D. M. Howard, "Digital waveguide mesh modelling of room acoustics: surround-sound, boundaries and plugin implementation," in *Proceedings of the COST G-6 Conference on Digital Audio Effects (DAFx)*, (Limerick, Ireland), Dec. 2001.

[34] J. Huopaniemi, L. Savioja, and M. Karjalainen, "Modeling of reflections and air absorption in acoustical spaces: a digital filter design approach," in *Proceedings of the IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, (New Paltz, New York), Oct. 1997.

[35] K. Lee and J. O. Smith, "Implementation of a highly diffusing 2-D digital waveguide mesh with a quadratic residue diffuser," in *Proceedings of the International Computer Music Conference*, (Miami), Nov. 2004.

[36] J. Laird, P. Masri, and N. Canagarajah, "Modelling diffusion at the boundary of a digital waveguide mesh," in *Proceedings of the International Computer Music Conference*, (Beijing, China), Oct. 1999.

[37] J. C. Strikwerda, *Finite Difference Schemes and Partial Differential Equations*. California, USA: Wadsworth, Brooks & Cole, 1989.

[38] A. Kelloniemi, D. T. Murphy, L. Savioja, and V. Välimäki, "Boundary conditions in a multi-dimensional digital waveguide mesh," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP'04)*, (Montreal, Canada), 2004.

[39] D. Botteldooren, "Finite-difference time-domain simulation of low-frequency room acoustic problems," *J. Acoust. Soc. Am.*, vol. 98, pp. 3302–3308, 1995.

[40] J. O. Smith, *Applications of Digital Signal Processing to Audio and Acoustics*, ch. Principles of Digital Waveguide Models of Musical Instruments, pp. 417–466. Kluwer Academic Publishers, 1998.

[41] S. Bilbao and J. O. Smith, "Finite difference schemes and digital waveguide networks for the wave equation: Stability, passivity, and numerical dispersion," *IEEE Trans. Speech and Audio Processing*, vol. 11, no. 3, pp. 255–266, 2003.

[42] M. Karjalainen and C. Erkut, "Digital waveguides versus finite difference structures: Equivalence and mixed modeling," *EURASIP Journal on Applied Signal Processing*, vol. 2004, no. 7, pp. 978–989, 2004.

[43] J. O. Smith, "On the equivalence of digital waveguide and finite difference time domain schemes," July 21, 2004. URL: `http://arxiv.org/abs/physics/0407032/`.

[44] B. M. Gibbs and D. K. Jones, "A simple image method for calculating the distribution of sound pressure levels within an enclosure," *Acustica*, vol. 26, pp. 24–32, 1972.

[45] J. B. Allen and D. A. Berkley, "Image method for efficiently simulating small-room acoustics," *J. Acoust. Soc. Am.*, vol. 65, no. 4, pp. 943–950, 1979.

[46] J. Borish, "Extension of the image model to arbitrary polyhedra," *J. Acoust. Soc. Am.*, vol. 75, no. 6, pp. 1827–1836, 1984.

[47] H. Lee and B. H. Lee, "An efficient algorithm for the image model technique," *Applied Acoustics*, vol. 24, pp. 87–115, 1988.

[48] U. Kristiansen, A. Krokstad, and T. Follestad, "Extending the image method for higher order reflections," *Applied Acoustics*, vol. 38, pp. 195–206, 1993.

[49] P. Svensson, R. I. Fred, and J. Vanderkooy, "An analytic secondary source model for edge diffraction impulse responses," *J. Acoust. Soc. Am.*, vol. 106, no. 5, pp. 2331–2344, 1999.

[50] R. Torres, U. P. Svensson, and M. Kleiner, "Computation of edge diffraction for more accurate room acoustics auralization," *J. Acoust. Soc. Am.*, vol. 109, no. 2, pp. 600–610, 2001.

[51] L. Savioja, J. Huopaniemi, T. Lokki, and R. Väänänen, "Creating interactive virtual acoustic environments," *J. Audio Eng. Soc.*, vol. 47, no. 9, pp. 675–705, 1999.

[52] A. Silzle, P. Novo, and H. Strauss, "IKA-SIM: A system to generate auditory virtual environments," in *Proc. 116th AES Convention*, (Berlin, Germany), 2004. Preprint 6016.

[53] A. Krokstad, S. Strøm, and S. Sørsdal, "Calculating the acoustical room impulse response by the use of a ray tracing technique," *J. Sound and Vibration*, vol. 8, no. 1, pp. 118–125, 1968.

[54] J.-J. Embrechts, "Sound field distribution using randomly traced sound ray techniques," *Acustica*, vol. 51, pp. 288–295, 1982.

[55] A. Kulowski, "Algorithic representation of of the ray tracing technique," *Applied Acoustics*, vol. 18, pp. 449–469, 1985.

[56] H. Lehnert, "Systematic errors of the ray-tracing algorithm," *Applied Acoustics*, vol. 38, pp. 207–221, 1993.

[57] F. Antonacci, M. Foco, A. Sarti, and S. Tubaro, "Real time modeling of acoustic propagation in complex environments," in *Proc. Int. Conf. Digital Audio Effects (DAFx'04)*, (Naples, Italy), pp. 274–279, 2004.

[58] A. LeBot and A. Bocquillet, "Comparison of an integral equation on energy and the ray-tracing technique in room acoustics," *J. Acoust. Soc. Am.*, vol. 108, no. 4, pp. 1732–1740, 2000.

[59] J. O. Smith III, *Physical Audio Signal Processing: Digital Waveguide Modeling of Musical Instruments and Audio Effects*. Available online at `http://ccrma.stanford.edu/~jos/pasp04/`, August 2004.

[60] W. G. Gardner, *Applications of digital signal processing to audio and acoustics*, ch. Reverberation algorithms, pp. 85–131. Kluwer Academic Publishers, 1998.

[61] M. R. Schroeder, "Natural-sounding artificial reverberation," *Journal of the Audio Engineering Society*, vol. 10, no. 3, pp. 219–223, 1962.

[62] J. A. Moorer, "About this reverberation business," *Computer Music Journal*, vol. 3, no. 2, pp. 13–28, 1979.

[63] W. G. Gardner, "The virtual acoustic room," Master's thesis, Massachusetts Institute of Technology, 1992. Available on-line at `http://www.harmony-central.com/Computer/Programming/virtual-acoustic-room.ps.gz`.

[64] J. Dattorro, "Effect design – part 1: Reverberator and other filters," *Journal of the Audio Engineering Society*, vol. 45, no. 9, pp. 660–684, 1997.

[65] G. S. Kendall, W. L. Martens, and S. L. Decker, *Current Directions in Computer Music Research*, ch. Spatial Reverberation: Discussion and Demonstration, pp. 65–87. MIT Press, 1989.

[66] M. Gerzon, "Synthetic stereo reverberation: Part one," *Studio Sound*, vol. 13, pp. 632–635, Dec 1971.

[67] M. Gerzon, "Synthetic stereo reverberation: Part two," *Studio Sound*, vol. 14, pp. 24–28, Jan 1972.

[68] J. Stautner and M. Puckette, "Designing multichannel reverberators," *Computer Music Journal*, vol. 6, no. 1, pp. 52–65, 1982.

[69] J.-M. Jot and A. Chaigne, "Digital delay networks for designing artificial reverberators," in *The 90th Convention of the Audio Engineering Society*, (Paris, France), preprint 3030, 1991.

[70] R. Väänänen, V. Välimäki, J. Huopaniemi, and M. Karjalainen, "Efficient and parametric reverberator for room acoustics modeling," in *Proceedings of the International Computer Music Conference*, (Thessaloniki, Greece), pp. 200–203, Sept. 1997.

[71] J. O. Smith and D. Rocchesso, "Connections between feedback delay networks and waveguide networks for digital reverberation," in *Proc. 1994 Int. Computer Music Conf.*, (Århus, Denmark), pp. 376–377, 1994.

[72] D. Rocchesso, *DAFX – Digital Audio Effects*, ch. Spatial Effects, pp. 137–200. John Wiley & Sons, 2002.

[73] U. Zölzer, N. Fliege, M. Schonle, and M. Schusdziara, "Multirate digital reverberation system," in *The 89th Convention of the Audio Engineering Society*, (Los Angeles, California, U.S.A.), preprint 2968, 1990.

[74] W. G. Gardner, "Efficient convolution without input-output delay," *Journal of the Audio Engineering Society*, vol. 43, pp. 127–136, Mar. 1995.

[75] U. Zölzer, *Digital Audio Signal Processing*. John Wiley & Sons, 1997.

[76] D. Griesinger, "Practical processsors and programs for digital reverberation," in *Proceedings of the Audio Engineering Society 7th International Conference*, (Toronto, Canada), pp. 187–195, May 1989.

[77] D. Rocchesso, "The ball within the box: a sound-processing metaphor," *Computer Music Journal*, vol. 19, no. 4, pp. 47–57, 1995.

[78] H. Nironen, *Diffuse Reflections in Room Acoustics Modelling*. Helsinki University of Technology, 2004. MSc Thesis.

[79] V. Pulkki and T. Lokki, "Visualization of edge diffraction," *Acoustics Research Letters Online (ARLO)*, vol. 4, pp. 118–123, Oct. 2003.

[80] B. Blesser, "An interdisciplinary synthesis of reverberation viewpoints," *Journal of the Audio Engineering Society*, vol. 49, pp. 867–903, Oct. 2001.

[81] M. Karjalainen and H. Järveläinen, "More about this reverberation science: Perceptually good late reverberation," in *Preprint of 111th AES Convention*, (New York, USA), 2001. Convention paper 5415.

[82] P. Huang and J. O. Smith, "Loudness-based display and analysis applied to artificial reverberation," Computer Music Association, Nov. 2004.

[83] J. Blauert, *Spatial Hearing. The Psychophysics of Human Sound Localization*. Cambridge, MA, USA: MIT Press, 1997.

[84] M. Kleiner, B.-I. Dalenbäck, and U. P. Svensson, "Auralization — An overview," *J. Audio Eng. Soc.*, vol. 41, no. 11, pp. 861–875, 1993.

[85] T. Lokki, *Physically-Based Auralization — Design, Implementation, and Evaluation*. Helsinki University of Technology, 2002. Doctoral Thesis.

[86] V. Pulkki, "Virtual sound source positioning using vector base amplitude panning," *J. Audio Eng. Soc.*, vol. 45, no. 6, pp. 456–466, 1997.

[87] M. A. Gerzon, "Panpot laws for multispeaker stereo," in *Preprint of 92th AES Convention*, (Vienna, Austria), 1992. Preprint 3309.

[88] A. J. Berkhout, D. de Vries, and P. Vogel, "Acoustic control by wave field synthesis," *J. Acoust. Soc. Am.*, vol. 93, no. 5, 1993.

[89] V. R. Algazi, R. O. Duda, and D. M. Thompson, "Motion-tracked binaural sound," in *Preprint of 116th AES Convention*, (Berlin, Germany), 2004. Convention paper 6015.

[90] M. Karjalainen, "BlockCompiler: Efficient simulation of acoustic and audio systems," in *Preprint of 114th AES Convention*, (Amsterdam), 2003. Convention paper 5756.