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First Order Digital Filters--An Audio Cookbook

Application Note AN-11 **by Christopher Moore**

Introduction

The simplest digital filters are first order FIR and IIR filters, singly or in cascade. While not as powerful as higher order filters, they are nevertheless of considerable interest in audio for bass and treble controls, DC offset removal, high pass filters, and low pass filters. Also appealing is the low computational burden that first order filters place on DSP resources.

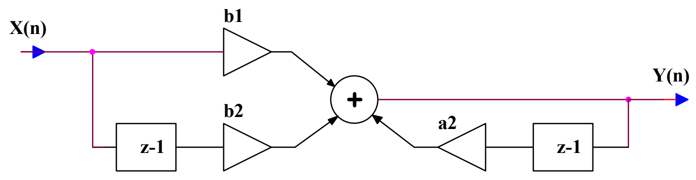
Despite the simplicity of the first order filter, the DSP literature is not forthcoming with a simple methodology that allows for cookbook design. The method that I present here appears in an article from 1978 by Ham and is based on relating a prototype analog filter to the desired digital filter using the technique of impulse invariance. See Mitra (1998), pages 425-431, for further information on this technique.

The system function of a first order discrete time filter is:

$$H(z) = \frac{b_1 + b_2 z^{-1}}{1 - a_2 z^{-1}}$$

Using the technique of impulse invariance, one ends up with exponential functions that let you map from the continuous time filter's corner frequencies directly to the coefficients of the discrete time (digital) filter.

The flow chart of a first order digital filter (Direct Form 1 structure) is shown below. Other structures, while yielding the same response, may be advantageous in some cases due to finite word length and other effects.



a2 is the coefficient of the pole feedback path. A plus sign in the tables indicates that the feedback path has a *positive* gain. If you are using the Matlab *freqz* function, you must enter -a2 to satisfy its sign convention for poles. The coefficient a1 is not shown on the flow chart as it relates to the gain of the output itself, which is always one.

To actually implement one of these filters, you will need a DSP environment in which to program a chip to carry out the indicated sample storage, multiplication, and accumulation.

Assembly language example (not optimized)
Motorola 56002

```
;Low pass 1st order filter
move   X:fsum,x0          ; source sample into x0
move   Y:B_1,y0           ; filter coef into y0
mpy    x0,y0,  a Y:B_2,y1 ; mpy source sample,
                          ;  get next coef
move   X:sumz0,x1         ; get z-1 delayed source
                          :  sample into x1
move   x0,X:sumz0         ; update delayed source
mac    x1,y1,a  Y:A_2,y1  ; mac 2nd sum term,
                          ; get next coef
move   x:intz0,x1         ; get z-1 output into x1
macr   y1,x1,a            ; macr 3rd and final term
move   a,X:intz0          ; update z-1 out register
move   a,X:fsum           ; update output
```

The values of b1 and b2 shown in the tables are simplified initial values, but the normalization factor norm was used in the *freqz* evaluation. Normalization was set to give unity gain at either DC or Fn, as appropriate. The normalization factor can be further modified according to signal dynamic range requirements. To incorporate normalization, scale b1 and b2 through multiplication by norm.

Each filter type is illustrated with a Matlab plot showing the digital filter response computed using Matlab's *freqz* function. Generally, the digital filter matches the prototype analog filter quite closely. However, at corner frequencies above about a quarter of the Nyquist frequency (5.6kHz here), the computed actual corner frequency begins to lag behind the target. Furthermore, as the signal frequency approaches Fn, the response of some of the filters deviates from the analog version by a few dB. Generally, the discrepancy can be ameliorated by the appropriate activation of a pole or zero of around +/-0.12.

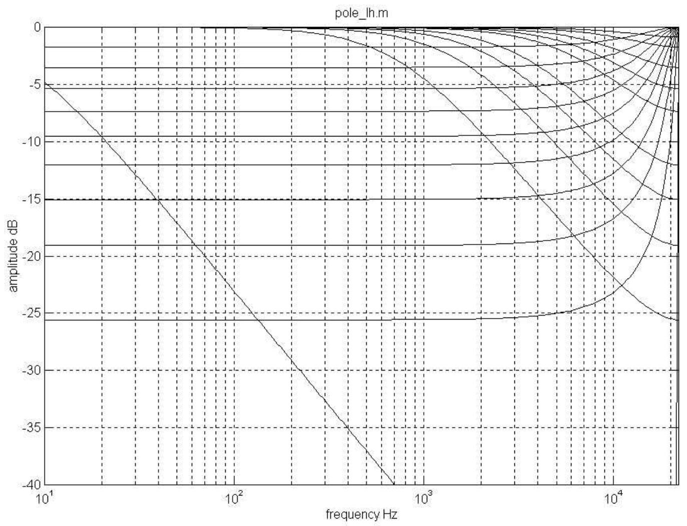
Definitions of Terms

Fs (sampling rate)	= 44.1kHz here, but can be any value
z^{-1} (unit delay of sampling)	= 1/Fs, seconds
Fn (Nyquist rate)	= Fs/2, 22.05kHz
f:	frequency, Hz
f1,f2:	filter corner frequency, Hz
a1, a2, b1, b2:	coefficients of the system function
abs():	absolute value function
e (base of natural logarithms)	= 2.718..
zero location:	-b2/b1
pole location:	a2

Pole

The first order pole by itself is capable of either low pass or high pass responses, as can be seen below. The low pass curves look potentially useful, but, by themselves, the high pass curves don't do much until pretty high frequencies. The high pass curves will come in handy to lift curves where the zero response doesn't quite behave.

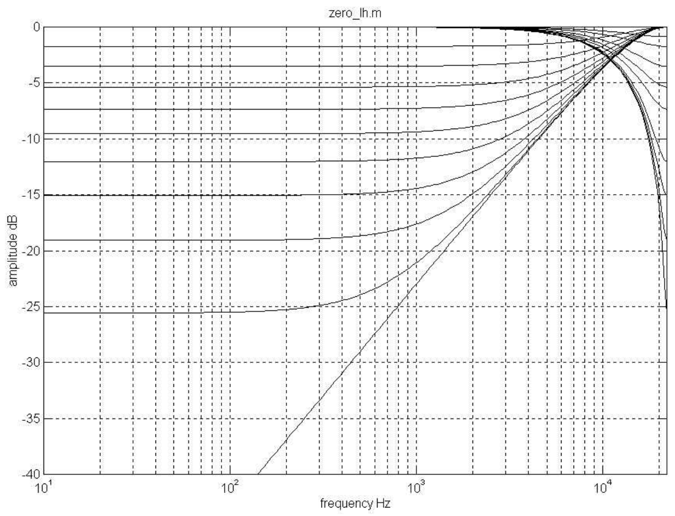
Pole (POLE_LH.M)				
b_1	b_2	norm	a_1	a_2
1	0	1/(1-abs(a2))	1	varied from 0 to +/- 0.999 in steps of 0.1



Zero

The first order zero by itself is also capable of either low pass or high pass responses. The high pass curves look potentially useful, but, by themselves, the low pass curves don't do much until pretty high frequencies. The low pass curves will come in handy to pull down curves where the pole response doesn't quite behave. It is very hard to look at these curves and deduce how to choose pole and zero responses that together will yield the desired filter.

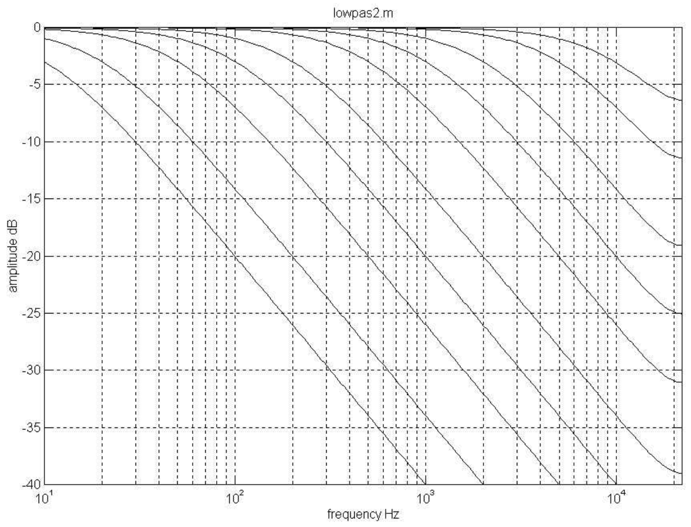
Zero (ZERO_LH.M)				
b_1	b_2	norm	a_1	a_2
-1	varied from 0 to +/-1.0 in steps of 0.1	abs(b1)+abs(b2)	1	0



Low Pass Filter

The low pass filter utilizes only the feedback portion of the filter to realize a pole that sets f1, the corner frequency, appropriately. In the curves below, f1 was set to 10, 20, 50, 100, 200, 500, 1k, 2k, 5k, and 10kHz. By enabling a zero at -0.12, you can add the extra attenuation needed to more closely match the continuous time filter at 10kHz and above. Furthermore, if additional roll off is required in the range of 4kHz and higher, the zero can be set even to -1.0, which gives zero output at Fn.

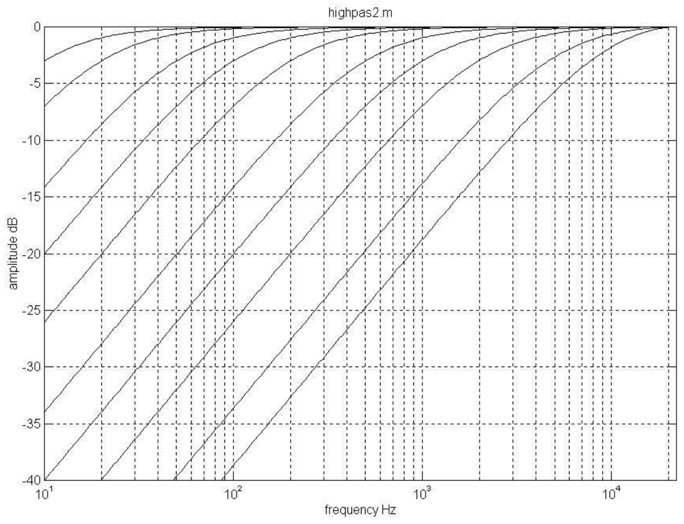
Low Pass (LOWPAS2.M)				
b_1	b_2	norm	a_1	a_2
1.0	0.12	(1-a2)/(b1+b2)	1	$e^{-\pi f_1 / F_N}$



High Pass Filter

The high pass filter utilizes the feed forward portion of the filter to realize a zero at DC and the feedback portion to create a pole that pulls the rising response down to a flat response. The response rises from zero at DC to a shelf at f1. This characteristic is useful for DC offset removal filters. In the curve below, f1 was set to 10, 20, 50, 100, 200, 500, 1k, 2k, 5k, and 10kHz.

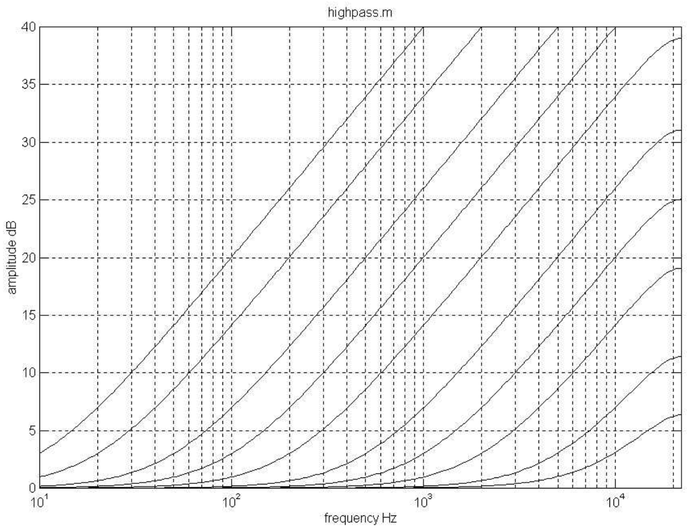
High Pass (HIGHPAS2.M)				
b_1	b_2	norm	a_1	a_2
1	-1	(1+a2)/2	1	$e^{-\pi f_1 / F_N}$



High Pass Filter with Low Frequency Shelf

The high pass filter utilizes only the feed forward portion of the filter to realize a shelf at low frequencies. The response begins rising off the shelf at f2. In the curves below, f2 was set to 10, 20, 50, 100, 200, 500, 1k, 2k, 5k, and 10kHz. By enabling a pole at -0.12, you can add the extra gain needed to more closely match the continuous time filter at 10kHz and above.

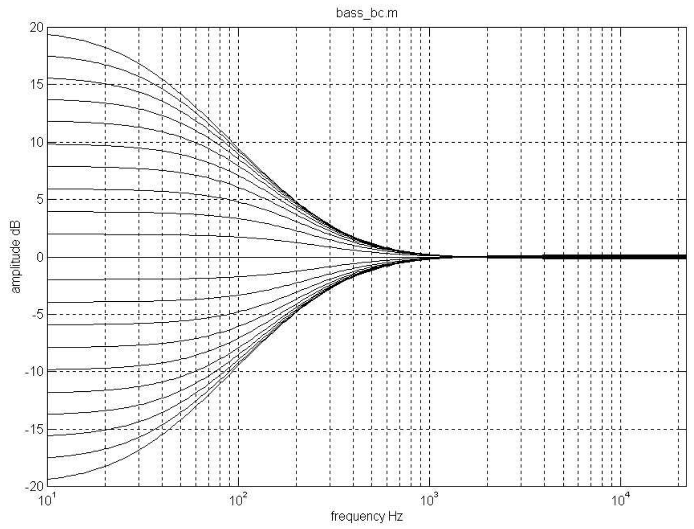
High Pass (HIGHPASS.M)				
b_1	b_2	norm	a_1	a_2
-1	$e^{-\pi f_2 / F_N}$	(1-a2)/abs(b1+b2)	1	-0.12



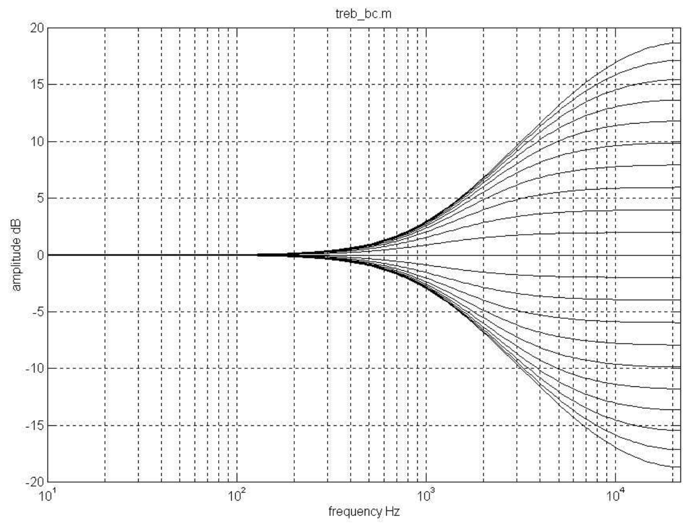
Shelf (bass, treble)

The shelf filter utilizes both feed forward and feed back sections. If f1 < f2, output falls from higher to lower level with increasing frequency, and response is suitable for bass boost or treble cut. If f2 < f1, output rises from lower to higher level with increasing frequency, and response is suitable for bass cut or treble boost. Shelf level difference is dB(f1/f2). In the bass and treble graphs below, one corner frequency was fixed (300Hz for the bass control and 1kHz for the treble control) and the other corner frequency was varied in log spaced steps over a range of 10:1. The Matlab file for the treble control is given on the last page.

Shelf (BASS_BC.M)				
b_1	b_2	norm	a_1	a_2
-1	$e^{-\pi f_2 / F_N}$	1	1	$-e^{-\pi f_1 / F_N}$



Shelf (TREB_BC.M)				
b_1	b_2	norm	a_1	a_2
-1	$e^{-\pi f_2 / F_N}$	(1-a2)/(b1+b2)	1	$e^{-\pi f_1 / F_N}$



Design notes

Even though the expressions used to compute the coefficients are quite simple, the computation should be mechanized if more than a few curves are required. I used Matlab, creating multiple files, one for each of the graphs seen here (the file names are shown in parantheses), but MathCAD or even a spreadsheet could also be used.

Example Matlab file, TREB_BC.M

```
%TREB_BC.M  Creates treble control with corner fre-
quency at 1kHz and sweeps from -20dB to +20dB in 20
steps
%
% revision date: March 3, 2001
%
% constants
Fs=44100;     % sampling rate
t=1/Fs;       % sampling interval
Fn=Fs/2;      % Nyquist frequency
minF=10;      % minimum frequency of analysis
maxF=Fs/2;    % maximum frequency of analysis
numPts=256;   % number of points of analysis
%
% create array of frequencies to use for freqz
Farray=logspace(log10(minF),log10(maxF),numPts);
%
% create arrays of frequencies in Hz to use for
sweeping the treble control
Fcarray1=[ 1000 1258 1584 1995 2512 3162 3981 5011
6310  7943  10000  1000 1000 1000 1000 1000 1000
1000  1000  1000   1000 ];
Fcarray2=[ 1000 1000 1000 1000 1000 1000 1000 1000
1000  1000   1000  1258 1584 1995 2512 3162 3981
5011  6310  7943  10000 ];
%
clf;     % clear graph
for A_index = 1: 21  % plot 21 times for 2dB steps
of boost and cut over +/- 20dB range
%
title ('treb_bc.m');    % do every iteration to get
correct appearance
xlabel ('frequency Hz');
ylabel ('amplitude dB');
axis([minF maxF -20 20 ]);
grid on;
%
f2=Fcarray2(A_index); % numerator (zero) freq
f1=Fcarray1(A_index); % denominator (pole) freq
%
%-----------------------------------------------
% compute the digital filter coefficients
%
L1= exp(-pi*f2/Fn); % numerator
K1= exp(-pi*f1/Fn); % denominator
%
b1=-1,   %numerator coefficient X(n)
b2= L1,  %numerator coefficient X(n-1)
a1= 1,   %denominator coefficient Y(n)
a2= K1,  %denominator coefficient Y(n-1)
normalize=(1-a2)/(b1+b2);  % normalize to AV=0dB at
DC
b1=normalize*b1;
b2=normalize*b2;
%
%define numerator and denominator of transfer func-
tion
  num=[ b1 b2   ], den=[ a1 -a2  ],
% num=[ Xn Xn-1 ], den=[ Yn Yn-1 ],
%
% fill array H with transfer function amplitude
[H,W]=freqz(num,den,Farray,Fs);
%
% plot digital filter magnitude response, log in x,
linear in dB in y
semilogx(Farray,20*log10(H),'w');
hold on; % Hold plot from prior to next case
%
end   % A_index loop
```

In conclusion—

Although I've presented a variety of equalization and filter curves here, the world of first order filters offers still more possibilities. The zero and the pole each can be located in two possible regions: between zero and +1 (DC), and between zero and -1 (Nyquist frequency). This gives four possible combinations, and, since there are an infinite number of possible locations, many more curves are possible.

Acknowledgements

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Bibliography

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