Maximally Diffusive Yet Efficient Feedback Delay Networks for Artificial Reverberation

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***Abstract*—Feedback delay networks are widely used for simulating the diffuse part of reverberation in a room. We present particular choices of the feedback coefficients, namely Galois sequences arranged in a circulant matrix, to produce a maximum echo density in the time response. These specific sets of coefficients give implementations having a low number of multipliers, and the resulting circuit can be efficiently pipelined. The resulting networks are compared with other efficient implementations.**

***Index Terms*— Acoustic signal processing, reverberation, sequences.**

I. Introduction

ARTIFICIAL reverberation has been an active field of research in audio signal processing for over 30 years [1]–[11], the purpose being to provide an effective simulation of the sound field as is given by actual rooms. In particular, several solutions have been proposed in the past for simulating the diffuse part of reverberation ("reverb"), i.e., the tail of the impulse response obtained by cutting the early 60–80 ms.

Most frequently, artificial reverberators are made of combinations of comb and allpass filters [1]–[4]. The main drawback of these structures is that they are difficult to parameterize because of the poor relationship between the system parameters and the physical quantities of a real room.

Another approach to reverberation is by means of waveguide filters [6]. With this technique, structures of arbitrary complexity can be built, and all the parameters can be assigned to physical quantities. On the other hand, fairly complicated (and computationally expensive) structures are needed in order to give a satisfactory approximation of the diffuse sound field in a room.

A computational structure in between the efficient comb/allpass networks and the general waveguide networks is the *feedback delay network* (FDN) [5], [7], [12], [13]. In this letter, we are proposing a special case of FDN, having optimal time-domain behavior and a low number of multipliers, which is based on circulant feedback delay networks (CFDN) [10], [11].

In Section II, we briefly describe the structure of FDN's and CFDN's. In Section III, a new realization based on Galois sequences is proposed. In Section IV, we suggest a possible implementation that exploits the nice properties of the structure. In Section V, we compare the proposed implementation with two other interesting solutions, namely the FDN proposed by Jot [14] and the FDN using a Hadamard feedback matrix.

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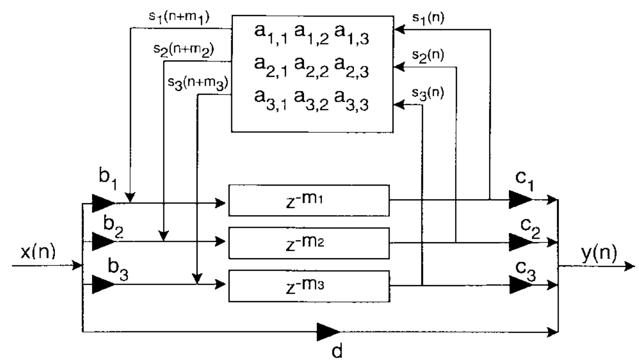

Fig. 1. Reference structure for a feedback delay network.

II. Background

An FDN is built using N delay lines, each having a length in seconds given by $\tau_i = m_i T$, where $T = 1/F_s$ is the sampling period. The complete FDN (see Fig. 1) is given by the following relations:

$$\begin{cases} y(n) = \sum_{i=1}^{N} c_i s_i(n) + dx(n) \\ s_i(n+m_i) = \sum_{j=1}^{N} a_{i,j} s_j(n) + b_i x(n) \end{cases} \tag{1}$$

where $s_i(n)$, $1 \le i \le N$, are the delay-line outputs at time sample n. The matrix $\mathbf{A} = [a_{i,j}]$ is called the "feedback matrix." The formulation of (1) represents a reference structure, in the sense that, with the appropriate choice of feedback matrix, it is a lossless structure. In practice, we must insert attenuation coefficients and filters in the feedback loop [7], [8].

In [10] and [11], CFDN's were proposed. For these structures, the feedback matrix is circulant [15], and the computation of the matrix–vector product in (1) can be performed in $O(N \log N)$ steps by using the fast Fourier transform (FFT) algorithm. Moreover, the matrix is completely specified by its first row, which can be found through an inverse FFT applied to the eigenvalues. It turns out that eigenvalues are useful for controlling the time–frequency properties of the reverberator. In [11], a procedure is also given for computing the delay-line lengths and the $\mathbf{b}$ and $\mathbf{c}$ coefficients starting from geometrical specifications of a room. Namely, the structure can be interpreted as a scattering object in a perfectly reflecting box, where each delay line is associated with the direction of propagation of sound waves to the object [16].

III. A Maximally Diffusive CFDN

In room acoustics, diffusors are gratings to be applied to the walls and the ceiling in order to improve the smoothness of reverb and distribute the sound energy uniformly in the whole enclosure [17]. By means of accurate selection of surface materials and geometry, it is possible to approximate a desirable scattering pattern in a certain frequency band [3], [18], [19]. One of the most desirable patterns turns out to be flat in all directions, thus corresponding to maximal diffusion.

In the physical interpretation of CFDN's [16], the feedback matrix represents the scattering properties of an object in a perfectly reflecting enclosure. For a maximally diffusive reverb, we want that any ray that is incident to the object be scattered in equal proportion along the various directions. In the time domain, this corresponds to a maximally dense impulse response. This is a highly desirable property for an artificial reverb [14], and might be obtained by choosing a sequence of equal-magnitude numbers for the first row of the circulant matrix. At the same time, we have to make sure that the structure remains stable, i.e., the matrix eigenvalues are within the unit circle [11], and this can be achieved by using Galois sequences [20] for the first row of the circulant feedback matrix.

For our purpose,[1] Galois sequences are defined as sequences of $N = 2^m - 1$ real numbers having magnitude $1/m$ and such that the discrete Fourier transform is constant and unit-magnitude, except for the DC component, which has magnitude $1/m$. This means that, for the feedback matrix, all the eigenvalues but one are on the unit circle. To bring the nonunitary eigenvalue on the unit circle we simply have to add a constant offset $\alpha \triangleq -[(m-1)/m(2^m-1)]$ to all the elements of the feedback matrix. With all the eigenvalues of the feedback matrix on the unit circle, the matrix is lossless [11]; hence, the frequency properties are controlled only by the loop filters and attenuation coefficients, and therefore no compensation is needed. Fig. 2 shows the impulse response of a maximally diffusive CFDN having delay lengths in samples $m_{1\cdots15}$ = [42, 29, 26, 23, 21, 19, 18, 17, 16, 15, 14, 13, 11, 9, 7] and, for any delay i, an attenuation coefficient $g_i = \rho^{m_i}$, with $\rho = 0.999$.

The fact that all the numbers of a Galois sequence have the same magnitude means that only $O(N)$ multiplications are needed to perform the matrix–vector multiplication. In the implementation suggested below, the number of multipliers can be further reduced to only one.

IV. Implementation

Experiments have shown that matrices of order larger than eight are needed to give a sufficiently dense, high-quality reverb. We present a practical implementation for $N = 15 = 2^4 - 1$. In the case under consideration, a Galois sequence is

$$\frac{[-1, 1, 1, 1, -1, 1, 1, -1, -1, 1, -1, 1, -1, -1, -1]}{4}. \tag{2}$$

By replacing 1 with 0 and -1 by $+1$, the sequence (2) can be obtained dividing by four the solution of the recursion

$$a_{k+4} = a_{k+1} + a_k, \qquad k = 0 \cdots 10 \tag{3}$$

where the addition is performed modulo 2 and the initial conditions are set to $a_0 = 1, a_1 = 0, a_2 = 0, a_3 = 0$ [20]. According to (2), the first element of the matrix–vector product $\mathbf{As}$ can be computed by the circuit of Fig. 3. The other elements are similarly computed. The expression $\alpha \sum_i s[i]$, represented by the lower path in Fig. 3, has to be computed only once for all the elements of the matrix–vector product. The computation of $(\mathbf{As})[i]$, for an arbitrary i, can be performed using the same hardware of Fig. 3, and a shift register containing the elements of the vector $\mathbf{s}$. For the ith element of the output vector, a shift is performed on the shift register, in order to bring the element $s[(i-1) \bmod N]$ in the first position, according to the circulant structure of the feedback matrix. Therefore, for this case with $N = 15$, for any output element, 15 two-input adds, one shift (multiply by 1/4), and an inverter, are required. The 15 two-input adds can be subdivided as follows: six adds for the adder *b* (Fig. 3), seven adds for the adder *c*, and two adds for the adder *a*. This number of additions corresponds to performing a multi-input add as a cascade of two-input adds. Whenever possible, an organization of two-input adders on a binary tree should be used to exploit parallelism and perform a n-input addition in $\Omega(\log n)$ steps. For example, the upper part of Fig. 3 can be organized according to Fig. 4. If additions are performed at the same rate as the shifts in the $\mathbf{s}$ sequence, a pipeline in the stages of the addition tree allows to complete the computation of the output vector $\mathbf{As}$ every N clock cycles, with a delay of $m_d = \log_2(N+1)$ cycles. The total number of adders to be used in the tree is $N - 1$. The total work W for a single matrix–vector product is given by the product of $N-1$ (adders) times $N + m_d$ pipeline stages, i.e., $W = 266$ for $N = 15$.

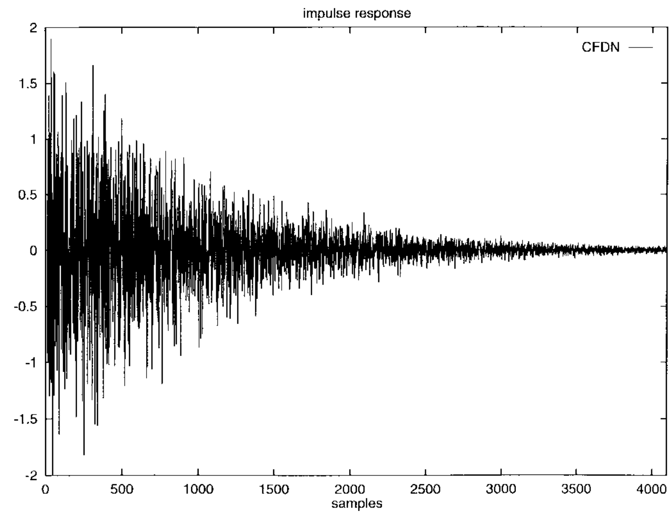

Fig. 2. Impulse response of a maximally diffusive CFDN, with $N = 15$.

[1] Since, for our application, it is crucial to have a unit-magnitude discrete Fourier transform, i.e., eigenvalues on the unit circle, we do not use the definition of Galois sequences as sequences of $+1$ and -1 [20].

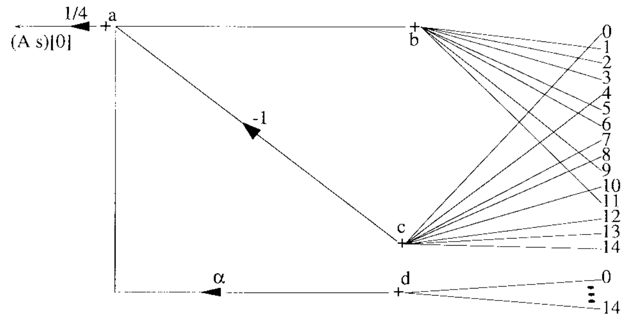

Fig. 3. First component of the matrix–vector computation for the maximally diffusive structure, with $N = 15$.

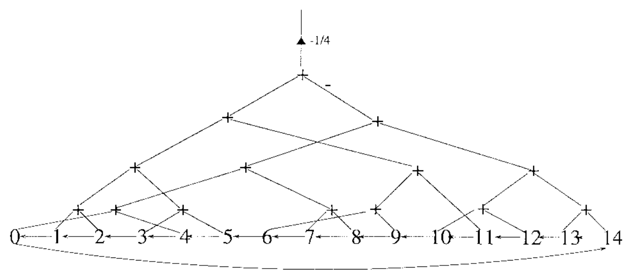

Fig. 4. Operations for the upper part of Fig. 3, organized in a binary tree.

In addition to the operations listed above, the sum of N numbers and the multiplication by α have to be computed once for the whole collection of N outputs. However, our experience in the simulation of room responses shows that this computation, represented by the lower part of Fig. 3, can often be omitted, since the difference in the output sound is not perceivable in most of the cases. Neglecting the contribution of the term $\alpha \sum_i s[i]$ corresponds to using a lossy matrix, where the loss is fairly small to be neglected.

We conclude this section by noting that the choice $N = 15$ is particularly attractive since the resulting circuit can be implemented with no multiplies at all, whenever the lower part of Fig. 3 is neglected, and the multiplication by 1/4 is replaced by a right shift.

V. Other Structures

Jot [14, p. 126] proposed a 16 × 16 unitary feedback matrix composed of 4 × 4 unitary blocks, each of them being the scattering matrix of a four-branches waveguide junction [9]. Such a matrix is maximally diffusive, since its values are all equal-magnitude, and it is lossless, since it is unitary. Moreover, the number of operations is fairly tractable, being about 4 × 16. However, such a matrix has not a circulant structure and, therefore, it does not admit the simple implementation of Figs. 3 and 4. Actually, we figured out a possible hard-wired implementation of the matrix–vector multiplication, using a fat tree having the vector elements at the leaves, 60 adders at the nodes, and 20 1-b shifters (see Fig. 5). The height of the tree is five, thus allowing five pipeline stages, and a total work $W = 300$. A drawback of this solution is the number of wires for connections, which is significantly larger than that of Fig. 4.

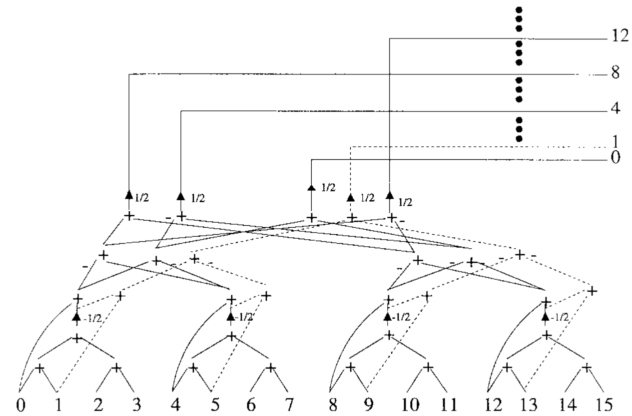

Fig. 5. Implementation of the matrix–vector multiplication using the 16 × 16 matrix proposed by Jot (a tree subset is shown).

Another good choice, suggested by Smith [21] for the feedback matrix of an FDN, is given by Hadamard matrices [15], which can be obtained recursively, for dimensions that are powers of 2, via Kronecker products. The benefits of such a choice are similar to the previous one, since the matrix is maximally diffusive and the matrix–vector product can be implemented recursively with no multiplies in $\Omega(N \log N)$ time steps. It has been shown that, up to high dimensions, the only Hadamard matrix that is circulant is 4 × 4, which is not enough for good reverb. Hence, in general, it is not possible to arrange the computation as nicely as in Fig. 4.

VI. Conclusion

A few feedback matrix structures are available to the designer of a maximally diffusive feedback delay network. To summarize, the Jot's and Hadamard matrices are the structures of choice within an FDN having order $N = 16$. On the other hand, choosing $N = 15$, we can take advantage of the CFDN metaphor [11] and of the simple implementation of Fig. 4.

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