CMPT 368: Lecture 11
Delay Effects

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FIR/Convolution

- Since the feedforward coefficient's of the FIR filter are the same as the non-zero elements of the impulse response, a general expression for the FIR filter's output can also be given by

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k),$$

where $h(\cdot)$ is the impulse responses and replaces the coefficients b_k.

- When the relation between the input and the output of the FIR filter is expressed in terms of the input and impulse response, we say the the output is obtained by *convolving* the sequences $x(n)$ and $h(n)$.

Visualizing the convolution sum

The convolution sum is given by

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k).$$

n	0	1	2	3	4	5
x(n)	1	1	0	0	0	0
h(n)	1	3	3	1	0	0
h(0)x(n-0)	1	1	0	0	0	0
h(1)x(n-1)		3	3	0	0	0
h(2)x(n-2)			3	3	0	0
h(3)x(n-3)				1	1	0
y(n)	1	4	6	4	1	0

Convolution

- Convolution is a technique that allows us to create a new sound by incorporating spectral characteristics of two other sounds.

- The operation of convolution performed on two signals in the time domain is equivalent to the multiplication of their spectrum in the frequency domain.

- That is, the convolution of two time domain signals $x(n)$ and $w(n)$, with Fourier transforms $X(\omega)$ and $W(\omega)$ respectively, is given by

$$Y(\omega) = X(\omega)W(\omega)$$

where $y(n)$, the result of the convolution, is obtained through the *inverse* Fourier Transform of $Y(\omega)$.

Time-Varying Delay Effects

Time-varying audio effects can be simulated using delay lines whose delay time can be varied on a sample-to-sample basis. Some examples include:

- Flanging
- Phasing
- Chorus
- Plucked String (Karplus Strong Algorithm)
- Artificial Reverberation

The Delay Line

- The delay line is an elementary functional unit which models *acoustic propagation delay*. It is a fundamental building block of both digital waveguide models and delay effects processors.
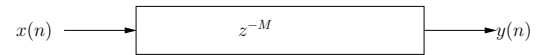
$$x(n) \longrightarrow \boxed{\quad z^{-M} \quad} \longrightarrow y(n)$$

Figure 1: The M-sample delay line.

- The function of a delay line is to introduce a time delay, corresponding to M samples between its input and output

$$y(n) = x(n - M), \quad n = 0, 1, 2, \ldots$$

The Simple Feedback Comb Filter

- What happens when we multiply the output of a delay line by a gain factor g then feed it back to the input?
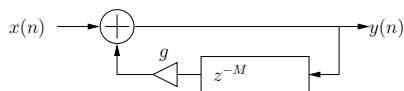

Figure 2: The signal flow diagram of a comb filter.

- The difference equation for this filter is given by

$$y(n) = x(n) + gy(n - M),$$

which is an IIR filter with a feedback coefficient g.

Comb Filter Impulse Response

- Let the M sample delay correspond to τ seconds.
- If the input to the filter is an impulse, $x(n) = \{1, 0, 0, \ldots\}$, the output will have an impulse of amplitude 1 at $t = 0$, followed τ seconds later by another impulse of amplitude g, followed again τ seconds later by another impulse of g^2, and so on.
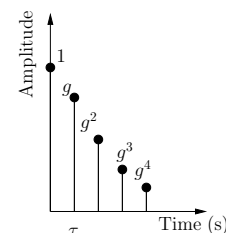

Figure 3: Impulse response for filter $y(n) = x(n) + gy(n - M)$.

- The impulse response is therefore a sequence of equally spaced impulses, each one with an amplitude a factor of g times that of the preceding impulse.

Comb Filter Charateristics

- Since the pulses are equally spaced in time at an interval equal to the loop time τ, it is periodic and will sound at the frequency $f_0 = 1/\tau$.

- The response decays exponentially as determined by the loop time and gain factor g.

- Notice that values of g nearest 1 yield the longest decay times.

Matlab example

```
fs = 44100;        %sampling rate
f0 = 220;          %desired fundamental
M = round(fs/f0);  %number of delay samples
g = 0.9;           %feedback gain

B = 1;
A = [1 zeros(1, M-1) g];

freqz(B, A);       %plot frequency response
```

Why is this a comb filter?

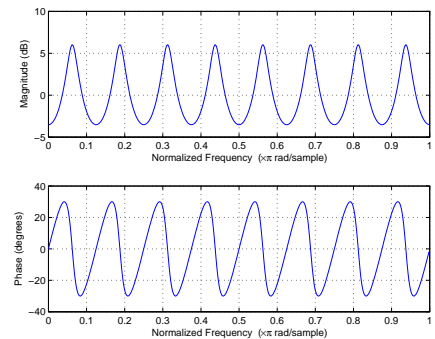- The comb filter is so called because its amplitude response resembles the teeth of a comb.

Figure 4: Frequency Response of a Comb Filter.

- The spacing between the maxima of the "teeth" is equal to the natural frequency.

- The depth of the minima and height of the maxima are set by the choice of g, where values closer to 1 yield more extreme maxima and minima.

General Comb Filter

- Consider now, adding to the filter a delay element which delays the input by M_1 samples, with some gain g_1.

- The general comb filter is given by the difference equation

$$y(n) = x(n) + g_1 x(n - M_1) - g_2 y(n - M_2)$$

where g_1 and g_2 are the feedforward and feedback coefficients, respectively.
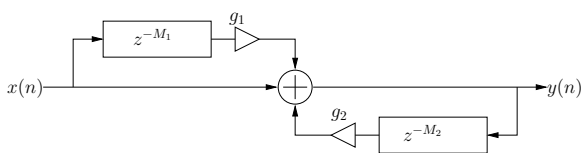

Figure 5: Signal flow diagram for digital comb filters.

Matlab Comb Filter Implementation

```
g1 = (0.5)^3;
g2 = (0.9)^5;
B = [1 0 0 g1];
A = [1 0 0 0 0 g2];
N = 1024;
x = zeros(N, 1);
x(1) = 1;
y = filter(B, A, x);
```
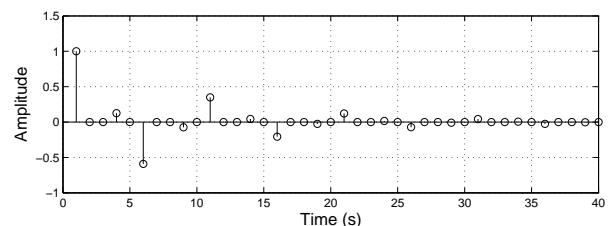

Figure 6: Comb Filter Impulse Response.

Flanging

- Flanging is a delay effect used in recording studios since the 1960s.

- Flanging creates a rapidly varying high-frequency sound by adding a signal to an image of itself that is delayed by a short, variable amount of time.

- Flanging was accomplished in analog studios by summing the outputs of two tape machines playing the same tape.

Flanging Process

- When the flange on one of the supply reals is touched lightly, it slows down and a delay is created between the two tape machines.

- The flange is then released while touching the flange on the other supply real, causing the delay to gradually disappear and then grow in the opposite direction. This process is repeated to the desired effect.
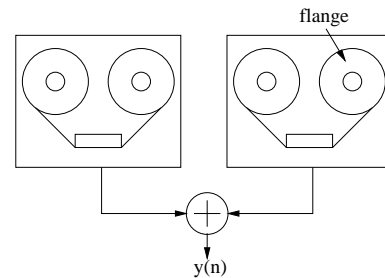

Figure 7: Two tape machines are used to produce the flanging effect.

Flange Comb Filter

- Since a delay between the two sources is needed, we should expect that a delay line will be used in the digital simulation.

- The flange simulation uses a *feedforward* comb filter, however, the delay is a function of time so that it can be swept, typically from a few milliseconds to 0, to produce the characteristic flange sound.

- The difference equation for the feedforward comb filter with a time-varying delay $M(n)$ is given by
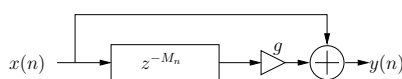
$$y(n) = x(n) + gx[n - M(n)].$$

Figure 8: A simple flanger.

The DEPTH parameter

- The feedforward coefficient g, also referred to as the DEPTH parameter, controls the proportion of the delayed signal in the output, determining the prominence of the flanging effect.

- The DEPTH parameter sets the amount of attenuation at the minima. It has a range from 0 to 1 (where 1 corresponds to maximum attenuation).
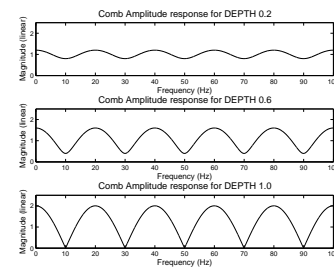

Figure 9: Depth parameter controlling notch attenuation.

- It is possible to allow for *dynamic flanging*, where the amount of flanging is proportional to the peak of the signal envelope passed through the flanger.

Delay parameter

- At any value of the delay $M(n)$, the minima, or *notches*, appear at frequencies that are **odd harmonics** of the inverse of twice the delay time.
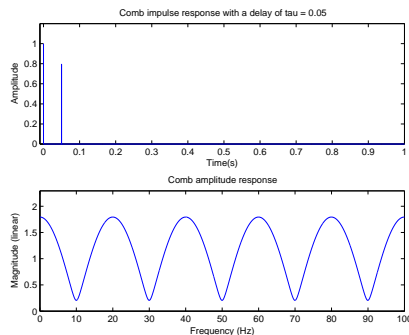

Figure 10: The impulse response and magnitude frequency response of a feedforward comb filter.

- Notches occur in the spectrum as a result of destructive interference (delaying a sine tone 180 degrees and summing with the original will cause the signal to disappear at the output).

Flanging Frequency Response

- Flangers produce their effect by dynamically changing the spectrum of the tone being processed.

- Flangers provide uniformly spaced notches. This can be considered non-ideal given that this will cause a discernible pitch to the effect.

- It can also cause a periodic tone to disappear completely through destructive interference. For this reason, flangers are best used with inharmonic (non-periodic) sounds (see drum demo).

Time-varying Delay

- Since the delay is varying over time, the number of delay samples M must also vary over time.

- This is typically handled by modulating $M(n)$ by a low-frequency oscillator (LFO). If the oscillator is sinusoidal, $M(n)$ varies according to

$$M(n) = M_0[1 + A\sin(2\pi f n T)],$$

where f is the rate or speed of the flanger in cycles per second, A is the "excursion" (maximum delay swing) and M_0 is the average delay length controlling the average notch density.

- For a successful flanging effect, M must change *smoothly* over time and therefore cannot have jumps in values associate with rounding to the nearest integer. We therefore must handle the fractional delay (can use linear interpolation).

Flanger summary

- We don't hear echo because the delays are too short (typically 1 - 10 ms)

- The delay has the effect of creating a series of notches in the frequency spectrum (comb filter).

- Notches occur in the spectrum as a result of destructive interference (delaying a sine tone 180 degrees and summing with the original will cause the signal to disappear at the output).

- The characteristic sound of a flanger results when these notches sweep up and down the frequency axis over time.

- The changing of the delay in the flanger creates some pitch modulation where the perceived pitch "warbles".

Phaser

- A phaser (or phase shifter) is a close cousin of the flanger, and in fact, the terms are often used interchangeably.

- The difference noted however, is that phasers modulate a set of non-uniformly spaced notches, whereas a flanger is a device which modulates uniformly spaced notches.

- To do this, the delay line of the flanger is replaced by a string of allpass filters.

Tapped Delay Line

- A *tap* refers to the extraction of the signal at a certain position within the delay-line.

- The tap may be interpolating or non-interpolating, and also may be scaled.

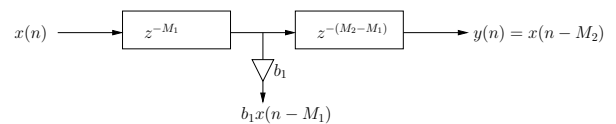- A tap implements a shorter delay line within a larger one.

Figure 11: A delay line *tapped* after a delay of M_1 samples.

Multi-Tap Delay Line Example

- Multi-Tapped delay lines efficiently simulate multiple echoes from the same source signal.
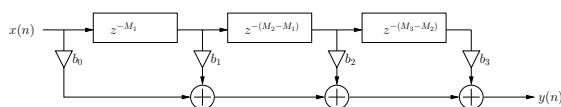

Figure 12: A multi-tapped delay with length M_3.

- In the above figure, the total delay line length is M_3 samples, and the internal taps are located at delays of M_1 and M_2 samples, respectively.

- The output signal is a linear combination of the input signal $x(n)$, the delay-line output $x(n - M_3)$, and the two tap signals $x(n - M_1)$ and $x(n - M_2)$.

- The difference equation is given by

$$y(n) = b_0 x(n) + b_1 x(n - M_1) + b_2 x(n - M_2) + b_3 x(n - M_3)$$

- *Convolution* is equivalent to tapping a delay line every sample and multiplying the output of each tap by the value of the impulse response for that time.

What is Chorus?

- A Chorus is produced when several musicians are playing simultaneously, but inevitably with small changes in the amplitudes and timings between each individual sound.

- It is very difficult (if not impossible) to play in precise synchronization—inevitably some randomness will occur among members of the ensemble.

- How do you create this effect using one single musician as a source?

- The chorus effect is a signal processing unit that changes the sound of a single source to a chorus by implementing the variability occurring when several sources attempt to play in unison.

Implementation of Chorus

- A chorus effect may be efficiently implemented using a multi-tap *fractional* delay line.

- The taps are not fixed and usually range from 10 to 50 ms.

- Their instantaneous delay may be determined using a random noise generator or, as in the flanger, a Low Frequency Oscillator (LFO).

- The chorus is similar to the flanger, only there is no feedback, and the delay times are typically longer (where a flanger was 1-10 ms, a chorus is about 10-50 ms).
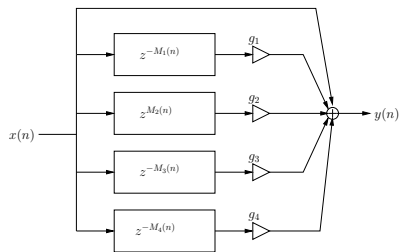

Figure 13: A bank for variable delay lines realize the chorus effect.

Reverberation

- Reverberation is produced naturally by the reflection of sounds off surfaces; it's effect on the overall sound that reaches the listener depends on the room or environment in which the sound is played.

- The amount and quality of reverb depends on
  - the volume and dimensions of the space,
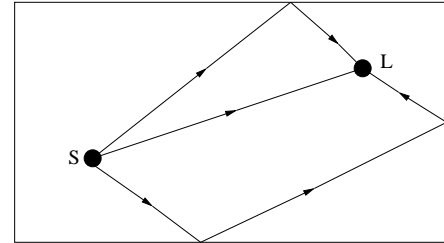 - and the type, shape and number of surfaces that the sound encounters.

Figure 14: Example reflection paths occurring between source and listener.

Reflections

- There are several paths the sound emanating from the source can take before reaching the listener, only one of which is *direct*.

- The listener receives many delayed images of the sound reflected from the walls, ceiling and floor of the room, which lengthen the time the listener hears the sound.

- The amplitude of the sound decreases at a rate inversely proportionate to the distance traveled.

- The sound is not only delayed, but it also decays. Reverb therefore tends to have a decaying amplitude envelope.

- Four physical measurements that effect the character of reverb are:

  1. Reverb time
  2. The frequency dependence of reverb time
  3. The time delay between the arrival of the direct sound and the first reflection
  4. The rate at which the echo density builds.

1.Reverb time

- The time required for a sound to die away to $1/1000$ (-60dB).

- Proportional to how long a listener will hear a sound, but depends also on other factors such as the amplitude of the original sound and the presence of other sounds.

- Depends on the volume of the room and the nature and number of its reflective surfaces.

  - Rooms with large volume tend to have long reverberation times.

  - With a constant volume, reverb time will decrease with an increase in surface area available for reflection.

  - Absorptivity of the surfaces: all materials absorb some acoustic energy. Hard, solid nonporous surfaces reflect efficiently whereas soft ones (such as curtains) absorb more substantially.

  - Roughness of the surfaces: if the surface is not perfectly flat, part of the sound is reflected and part is dispersed in other directions.

2. The frequency dependence of reverb time.

- Reverb time is not uniform over the range of audible frequencies. In a well designed concert hall, the low frequencies are the last to fade.

- Absorptive materials tend to reflect low-frequency sounds better than high ones.

- Efficient reflectors (such as marble) however, reflect sounds of all frequencies with nearly equal efficiency.

- With small solid objects, the efficiency and the direction of reflection are both dependent of frequency. This causes frequency-dependent dispersion and hence a major alteration of the waveform of a sound.

3. Time delay between the direct sound and the first reflection.

- A long delay (> 50 ms) can result in distinct echoes

- A short delay (< 5 ms) contributes to the listener's perception that the space is small

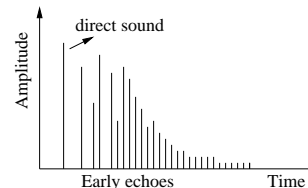- A delay between 10 and 20 ms is found in most good halls.

Figure 15: An example impulse response showing direction sound, early reflections, and exponential decay of the sound.

4. The rate at which the echo density builds

- After the initial reflection, the rate at which the echoes reach the listener begins to increase rapidly.

- A listener can distinguish differences in **echo density** up to a density of 1 echo/ms.

- The amount of time required to reach this threshold influences the character of the reverberation. In a good situation, this is typically 100 ms.

- This time is roughly proportional to the square root of the volume of a room, so that small spaces are characterized by a rapid buildup of echo density.

Digital Reverb

- An digital reverberator is a filter designed to have an impulse response emulating the impulse response of the space being simulated.

- Ideally, a digital simulation permits control over the parameters that determine the character of the reverberation.

- Delay lines can be used to simulate the travel time of the indirect sound (reflections) by delaying the sound by the appropriate length of time.

- We can simulate several different amounts of delay of the same signal using a single multi-tapped circular delay line, with a length suited for the longest required delay.

Implementation

- Consider a bank of Feedback Comb filters where each filter has a difference equation

$$y(n) = x(n) + gy(n - M).$$

- The response decays exponentially as determined by the loop time M and the coefficient g.

- To obtain a desired reverberation time, g is usually approximated, given the loop time τ, by

$$g = 0.001^{\tau/T_{60}}$$

where 0.001 is the level of the signal after the reverb time (the -60dB point) and T_{60} is the reverb time.

All-pass filters

- Unlike a comb filter, the all-pass filter passes signal of all frequencies equally. That is, the amplitudes of frequency components are not changed by the filter.

- The all-pass filter however, has substantial effect on the phase of individual signal components, that is, the time it takes for frequency components to get through the filter. This makes it ideal for modelling frequency *dispersion*.

- The effect is most audible on the transient response: during the attack or decay of a sound.

- The difference equation for the all-pass filter is given by

$$y(n) = -gx(n) + x(n - M) + gy(n - M).$$

Figure 16: Impulse and amplitude response of an all pass filter.

Network of unit reverberators

Parallel Connection

- When unit reverberators are connected in **parallel**, their impulse responses **add**.

- The total number of pulses produced is the **sum** of the pulses produced by the individual units.

Series Connection

- When placed in **series** (cascade), the impulse response of one unit triggers the response of the next, producing a much denser response.

- The number of pulses produced is the **product** of the number of pulses produced by the individual units.
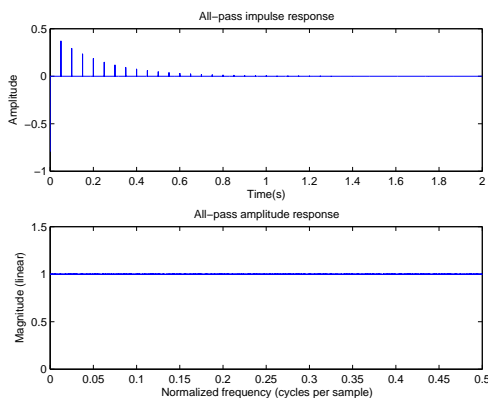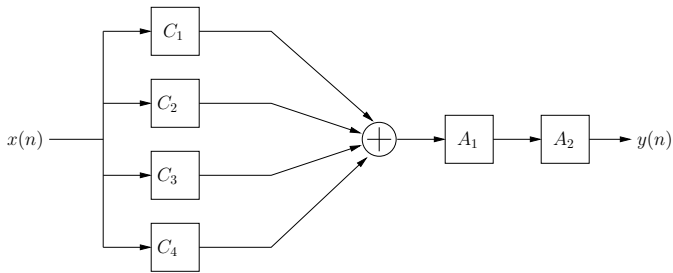
Two topolgies by Schroeder

Figure 17: Digital reverberator using parallel comb filters and series allpass filters.
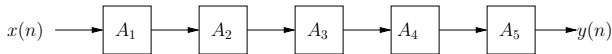

Figure 18: Digital reverberator using series allpass filters.

Choosing control parameter values

- Choose loop times that are relative prime to each other so the decay is smooth.

  – If the delay line loop times have common divisors, pulses will coincide, producing increased amplitude resulting in distinct echoes and an audible frequency bias.

- More than two unit reverberators are typically used in a design.

- Shorter loop times are used to simulate smaller spaces. In a smaller room, the first reflections will arrive sooner and the echo density will increase more rapidly.

Element	Reverb time	Loop time
C_1	RVT	29.7 ms
C_2	RVT	37.1 ms
C_3	RVT	41.1 ms
C_4	RVT	43.7 ms
A_1	5.0 ms	96.83 ms
A_2	1.7 ms	32.92 ms

Table 1: Parameters for a Schroeder Reverberator simulating a medium-sized concert hall (RVT indicates the reverberation time for the overall unit.)